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MAGNETIC FIELD GROWTH IN GALACTIC DISCS OF VARIOUS THICKNESS

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Abstract. A large number of galaxies have regular magnetic fields of several microgauss. Their evolution is described usnig the dynamo mechanism. It is based on helicity of the turbulent motions in the interstellar medium and differential rotation of the object. The process is described by mean field dynamo equation which is quite difficult to be solve both analytically and numerically. So different models taking into account geometrical shape of the object are used. One of the most popular approaches is connected with thin disc approximation which replaces the z-derivatives by algebraic expressions. However, as for discs with large half-thickness it does not give proper results. So we consider the thick disc model which can be useful for the objects with quite large half-thickness. Here we give the estimates of the magnetic field growth rate and check them numerically. It has been shown that for thick disc it is much more difficult to obtain the growing field. One of the most interesting questions (which has not been studied before) is connected with phase diagrams for the magnetic field. We have found a stable point and describe the typical phase trajectories leading to it

1. INTRODUCTION

A wide range of galaxies have large-scale magnetic fields of several microgauss (Beck et al 1996). First observational evidence was connected with studies of the cosmic rays: trajectories of the charged particles change under the influence of the magnetic fields. Fermi in his pioneer work has made estimates that are quite close to the values which are used now (Fermi 1949). Another important method is connected with studies of the synchrotron emission spectra which changes because of the magnetic field in the interstellar medium (Ginzburg 1959). However, today most of the observational studies of the galactic magnetic fields are done using Faraday rotation measurements (Manchester 1972). The polarized radiowaves pass through magnetized medium and the polarization plane rotates using the large-scale magnetic field as an axis. As for our Galaxy, usually pulsars are used as a source of the polarized radiowaves. Now there are more than 1000 objects that allow us to study the field structure in detail (Andreasyan et al. 2020).

Theoretically, the magnetic field generation is based on the dynamo mechanism (Beck 1996; Arshakian 2009). It is connected with helicity of the turbulent motions (which is also called alpha-effect) and differential rotation. They are described by mean field dynamo equation (Krause & Raedler 1980). It is connected with a three-dimensional vector problem, that is quite difficult. Analytical methods are very complicated and numerical approaches are connected with quite large computational resources. So, for different astrophysical objects the specific models are used. They take into account their symmetry and reduce the equations to quasi-two-dimensional systems.

As for galaxies, the thin disc model (Moss 1995) is widely used. It assumes specific simple vertical structure of the magnetic field, that allows to change the corresponding second derivatives by the algebraic expressions. So, we assume the equations for two components of the magnetic field which depend on one (rarely two) spatial coordinates. The thin disc model is widely used in studying galactic magnetism and gives an opportunity to find the field structure for a wide range of concrete objects. It is interesting to say that this model is also used while studying magnetic fields of accretion discs.

However, there are a large number of galaxies where the half-thickness of the disc and its radius are quite comparable. Especially it is connected with the outer parts of the galaxy, where the disc can be thicker than 1 kpc (Mikhailov et al 2014). So, the thin disc model seems to be unapplicable. It is necessary to take different assumptions. In this paper we present a specific approach which considers the magnetic field as a sum of toroidal component (it is found in implicit form) and poloidal field that is calculated as a curl of the azimuthal part of the vector potential (Deinzer et al 1993; Mikhailov & Pushkarev 2021; Mikhailov & Pashentseva 2022). We take a system of equations for the field components which depend on distance from the rotation axis and distance from the equatorial plane (so-called rz-model). The typical time dependeces of the field are described.

2. MEAN FIELD EQUATION

If we average the magnetic field in typical turbulent cells (as for galaxies, they have typical lengthscales of $50-100$kpc, we shall obtain the Steenbeck – Krause – Raedler equation (Krause & Raedler 1980):

$$\frac{\partial \mathbf{B}}{\partial t} = \mathrm{curl}(\alpha \mathbf{B}) + \mathrm{curl}(\mathbf{V} \times \mathbf{B}) + \nu \Delta \mathbf{B};$$

where $\mathbf{V}$ is the large-scale velocity (usually connected with galaxy rotation), α is the helicity of the turbulent motions. As for the galaxies, we usually can say that $\mathbf{V} = r\Omega \mathbf{e}_\varphi$. For the alpha-effect we can take $\alpha = \frac{\Omega l^2 z}{h^2}$, where l is the typical turbulence lengthscale and h is the half-thickness of the disc (Arshakian et al 2009).

The magnetic field cannot grow infinitely, so we should use so-called equipartition value defined as $B^* = 2v\sqrt{\pi\rho}$. The alpha-effect should be calculated using by the nonlinear formula $\alpha \sim (1 - \frac{B^2}{B^{*2}})$. This leads to nonlinear saturation and simple attractors for the magnetic field (Arshakian et al 2009).

3. THIN DISC MODEL

If we take thin disc model, we can neglect the z-component of the field. Its vertical derivative can be obtained using solenoidality condition. As for the main part of the magnetic field, we can take the approximate law $\mathbf{B}(r,z,t) = \mathbf{B}(r,0,t)\cos\left(\frac{\pi z}{2h}\right)$. So in the equipartition plane the second vertical derivatives of the field can be $\frac{\partial^2 \mathbf{B}}{\partial z^2} = -\frac{\pi^2 \mathbf{B}}{4h^2}$.

The equations in cylindrical coordinates and dimensionless units can be written as:

$$\frac{\partial B_r}{\partial t} = -R_\alpha B_\varphi (1 - B_r^2 - B_\varphi^2) - \frac{\pi^2}{4} B_r + \lambda^2 \left(\frac{\partial^2 B_r}{\partial r^2} + \frac{1}{r}\frac{\partial B_r}{\partial r} - \frac{B_r}{r^2} \right);$$

$$\frac{\partial B_\varphi}{\partial t} = R_\omega B_r - \frac{\pi^2}{4} B_r + \lambda^2 \left(\frac{\partial^2 B_\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial B_\varphi}{\partial r} - \frac{B_\varphi}{r^2} \right);$$

where R_α is connected with alpha-effect, R_ω shows differential rotation and λ – turbulent diffusivity.

It can be obtained that the magnetic field evolution is described by the dynamo number which is constructed as $D = R_\alpha R_\omega$. The field can grow for cases $D > D_{cr}$, where D_{cr} is the critical dynamo number. Using eigenvalue formulation of the problem (Mikhailov 2020), it can be obtained that $D_{cr} \approx 6..7$. The magnetic field growth rate can be obtained as $\gamma = -\frac{\pi^2}{4} + \sqrt{R_\alpha R_\omega}$.

The numerical solution for the magnetic field evolution for different D is shown on figure 1. It can be seen, that the magnetic field for small $D = 6$ really decays, and for larger D it grows exponentially. It can also be seen that for smaller $D = 12$ the growth rate is smaller than for larger $D = 18$.

4. THICK DISC MODEL

As for the magnetic fields in thick discs (Mikhailov & Pushkarev 2021; Mikhailov & Pashentseva 2022), or for the peripherical parts, we should take another assumptions. The field can be presented in the following form:

$$\mathbf{B} = \mathrm{curl}(A\mathbf{e}_\varphi) + B\mathbf{e}_\varphi.$$

The equations for the magnetic field will be:

$$\frac{\partial A}{\partial t} = R_\alpha z B(1 - B^2) + \lambda^2 \left(\frac{\partial^2 A}{\partial r^2} + \frac{1}{r}\frac{\partial A}{\partial r} - \frac{A}{r^2} \right);$$

$$\frac{\partial B}{\partial t} = R_\omega \frac{\partial A}{\partial z} + \lambda^2 \left(\frac{\partial^2 B}{\partial r^2} + \frac{1}{r}\frac{\partial B}{\partial r} - \frac{B}{r^2} \right).$$

The magnetic field growth rate can be obtained using eigenvalue formulation (Mikhailov & Pashentseva 2023) as

$$\gamma = -\frac{\pi^2}{4} + \frac{3}{4}\sqrt{R_\alpha R_\omega}.$$

So, the field can grow if $\gamma > 0$, and if $D > D_{cr}$, where $D_{cr} \approx 13$.

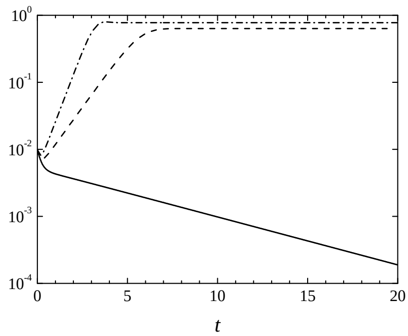

Figure 1: Evolution of the magnetic field in thin disc model. Solid line shows $D = 6$, dashed line – $D = 12$, dot-dashed line – $D = 18$.

The field evolution for this case is shown on figure 2. We can see, that for $D = 12$ the magnetic field decays, contrary to the thin disc model. We can also see that the magnetic field growth rate here is smaller (Mikhailov & Pushkarev 2021).

It is also interesting to study the evolution of $\frac{\partial A}{\partial z}$. This value characterizes the radial magnetic field. It is presented on figure 3. We can see, that for cases $D = 6$ and $D = 12$ the field decays, and for $D = 18$ it grows. Also it is worth to mention that the radial field is much smaller than the azimuthal one.

It is also interesting to study phase diagram of the system. The nonlinearity describes simple attractor with a stable point. Using different initial conditions (figure 4) we have found the typical stable solution for $D = 20$, corresponding to $\frac{\partial A}{\partial z} = 0.24$, $B = 1.01$.

5. CONCLUSION

We have studied the magnetic field evolution using different approaches. First of all, we have studied it using the thin disc model. After that, the thick disc model have been studied. It is interesting that for the thick disc it is much more difficult to generate the field. Also we have studied the evolution of the $\frac{\partial A}{\partial z}$, which characterizes the radial magnetic field. The phase trajectories on the plane $\left(\frac{\partial A}{\partial z}, B\right)$ have been studied, too. We have found the typical stable points for this case.

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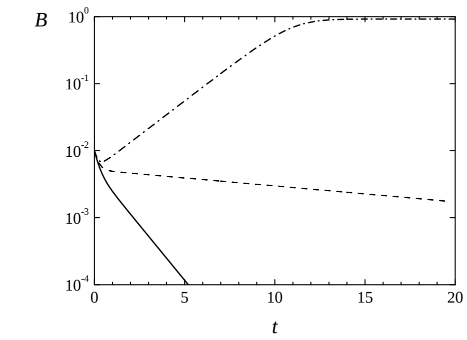

Figure 2: Evolution of derivative of vector potential component $\frac{\partial A}{\partial z}$ in thick disc model. Solid line shows $D = 6$, dashed line – $D = 12$, dot-dashed line – $D = 18$.

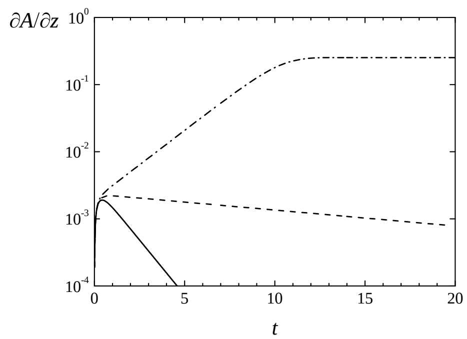

Figure 3: Evolution of derivative of azimuthal magnetic field B in thick disc model. Solid line shows $D = 6$, dashed line – $D = 12$, dot-dashed line – $D = 18$.

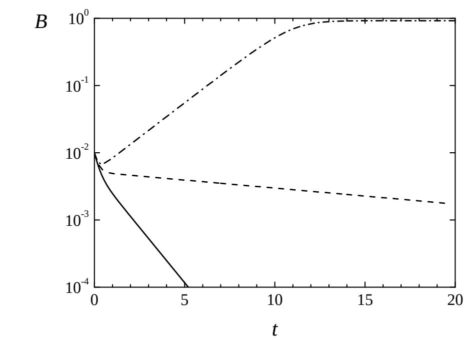

Figure 4: Phase diagram for thick disc model for $D = 20$. Different curves correspond to different initial conditions

References

Beck, R., Brandenburg, A., Moss D., Shukurov, A., Sokoloff, D.: 1996, *Ann.Rev.Astron.Astrophys.*, **34**, 155, doi: 10.1146/annurev.astro.34.1.155

Fermi, E.: 1949, *Phys. Rev.*, **75**, 1169, doi: 10.1103/PhysRev.75.1169

Ginzburg V.: 1959, *Symposium International Astronomical Union*, **9**, 589.

Manchester, R.: 1972, *The Astrophysical Journal*, **172**, 43, doi: 10.1086/151326

Andreasyan, R.R., Mikhailov, E.A., Andreasyan, H.R.: 2020, *Astronomy Reports*, **62**, 189, doi:10.1134/S1063772920030014

Arshakian, T., Beck, R., Krause, M., Sokoloff, D.: 2014, *Astronomy and Astrophysics*, **494**, 21, doi: 10.1051/0004-6361:200810964

Krause, F., Raedler, K.-H. Mean-field electrodynamics and dynami theory. Berlin, 1980.

Moss, D. *Monthly Notices of the Royal Astronomical Society*, **275**, 191, doi: 10.1093/mnras/275.1.191

Mikhailov, E., Kasparova, A., Moss, D., Beck, R., Sokoloff, D., Zasov, A. 2014: *Astronomy and Astrophysics*, **568**, A66. DOI:10.1051/0004-6361/201323341

Deinzer, W., Grosser, H., Schmitt, D. 1993, *Astronomy and Astrophysics*, **273**, 405.

Mikhailov, E., Pashentseva, M.: 2022, *Moscow University Physics Bulletin*, **77**, 741, doi:10.3103/S0027134922050101

Mikhailov, E., Pushkarev, V.: 2021, *Research in Astronomy and Astrophysics*, **21**, 056, doi: 10.1088/1674-4527/21/3/056

Mikhailov, E.: 2020, *Magnetohydrodynamics*, **56**, 403, doi: 10.22364/mhd.56.4.5

Mikhailov, E., Pashentseva, M.: 2023, *Mathematics*, **11**, 3106, doi: 10.3390/math11143106