Publ. Astron. Obs. Belgrade No. 107 (2025), 21 - 28 *Invited Lecture*
http://doi.org/10.69646/14sbac03p

ASTROPHYSICAL APPLICATIONS OF STARK BROADENING OF SINGLY IONIZED PALADIUM SPECTRAL LINES

M. D. CHRISTOVA[1] and M. S. DIMITRIJEVIĆ[2,3]
[1] *Department of Applied Physics, Technical University of Sofia, 1000 Sofia, Bulgaria*
E–mail: mchristo@tu-sofia.bg
[2] *Astronomical Observatory, Volgina 7, 11060 Belgrade, Serbia*
[3] *LERMA, Observatoire de Paris, Université PSL, CNRS, Sorbonne Université, F-92190 Meudon, France*

Abstract. We review our recently published work on the Stark broadening of Pd II spectral lines, calculated by employing the Modifiied semiempirical theory, which includes the applications of obtained results for the investigation of the influence of Stark broadening in the atmospheres of A type stars, DB and DO white dwarfs. The influence of Stark broadening in stellar atmospheres is discussed including the previously published Stark width results (calculated with the help of the same method) for spectral lines of other chemical elements. Discussion on the astrophysical applications is included. As an application of obtained results, Stark width behaviour with temperature of Pd II spectral lines is also analyzed.

1. INTRODUCTION

According to the literature, palladium spectral lines are detected in spectra of different types of stars. Biémont *et al.* (1982) report the presence of neutral palladium in solar spectrum. In very metal poor red giants HD 107 752, HD110 184, HD85 773, HD23 798, and BD+6^o648 are found Pd I spectral lines by Aoki *et al.* (2017). Adelman *et al.* (1979) present neutral palladium spectra for peculiar A star γ Equulei. Singly ionized palladium spectral lines are observed in the spectra of HgMn star χ Lupi by Lundberg *et al.* (1996) and in a post-asymptotic giant branch star of spectral type B8 III in the globular cluster 47 Tucanae (NGC104) by Dixon *et al.* (2021). This is an indication that experimental and calculated spectroscopic results for palladium lines are of interest for reliable analysis and stellar atmosphere modeling of these objects. Particularly, for hot stars, for Hg-Mn stars, A and late B type stars, and white dwarfs, Stark broadening data are needed.

In this study, we inform shortly about our work on Stark broadening of Pd II pectral lines and the influence of this broadening mechanism in stellar atmospheres spectra(Dimitrijević and Christova, 2024). Additionally, temperature dependence of Stark widths of Pd II spectral lines is discussed. The studied lines belong to 5s^{2}F - 5p^{2}G^o, 5s^{4}F - 5p^{4}D^o, 5s^{4}F - 5p^{4}G^o and 5s^{4}F - 5p^{4}F^o multiplets. The table with calculated Stark widths could be found in Dimitrijević and Christova (2024). The temperature interval covers values from 5 000 K to 160 000 K and the perturber density is 10^{17} cm-3. The modified semiempirical method (MSE) of Dimitrijević and Konjević (1980) is applied for 47 spectral lines of singly charged palladium ion.

Illustration of the importance of Stark broadening of spectral lines for the modelling of stellar atmospheres of DB and DO white dwarfs, as well as of A type star is also presented.

In the discussion of the influence of Stark broadening in stellar atmospheres, our previously published Stark width results (calculated with the help of the MSE method) for spectral lines of other chemical elements: Lu II (Dimitrijević and Christova, 2021), Si II (Dimitrijević *et al.*, 2023), Al IV (Dimitrijević and Christova, 2023), Zn III (Dimitrijević and Christova, 2022), *etc.* are also commented. For all these elements, including Pd II, we found in literature the need of Stark broadening data for the investigation, analysis and modeling of stellar plasma, which was our prncipal reason for investigations. These elements play a key role in nucleosynthesis mechanisms and could provide valuable interpretation of such processes. For example, for elements heavier than iron, nucleosynthesis is governed by two leading caption-neutron processes, s-process, and r-process. The difference is in time scale in comparison to the half-life of beta decay. The s-process is ongoing at a slower rate, while r-process occurs at a rapid rate. The r-elements formation is not well understood. An outstanding example of the information on the r-process signature is uranium-rich metal-poor star CS 31082-001 (Siqueira*et al.*, 2013), where Lu spectral lines are measured in the near-UV HST/STIS and optical UVES spectra. Since CS 31082-001 is an extremely metal poor star (EMP) with large r-element excesses, adequate abundance interpretation could help r-nucleosynthesis understanding. Lutetium belongs to rare earth elements (REE) and the main part of the line spectrum is in UV diapason. According to Roederer *et al.* (2012), the spectroscopic observations from Goddard High Resolution Spectrograph (GHRS) and Space Telescope Imaging Spectrograph (STIS) on board of the Hubble Space Telescope (HST) for spectral lines of heavy elements indicate that almost all stars contain traces, at least of chemical elements heavier than iron. Lu is one of these elements which determine a key role for understanding neutron-capture nucleosynthesis mechanisms and could be observed mainly in near ultraviolet. It is reported that rare earth elements, as lutetium, are detected in metal poor red giant stars as first ions. The reliable modeling of metal poor stars atmospheres requires huge atomic and molecular data and Stark broadening parameters, also. The lack of experimental and theoretical Lu II Stark broadening data in the literature, was the motivation for our investigation.

In contrast to lutetium, silicon and aluminum are one of the richly populated elements in the universe, the sixth and twelfth most common elements, respectively. Silicon ions are very important for investigations of stellar and solar plasmas. One of the earlier papers (Peytremann, 1972) presents various cosmic light sources that contain silicon atoms and ions as emitters. He observed many singly charged ion lines in the atmospheres of A, B and O type stars, and white dwarfs, also. Lanz *et al.* (1988) report that the spectrum of Si II is dominant for stars from A0 to B3 spectral type (effective temperatures from 10,000 to 20,000 K) where strong visible spectral lines are observed. At these plasma conditions, broadening due to collisions with electrons is the main pressure broadening mechanism. Solar photosphere study is performed by Shi *et al.* (2008) using ionized silicon lines. The widths of such lines enable the diversity determination of supernovae Ia by Arsenijević*et al.* (2008). Prominent Si II lines take part in the emission spectra of the first helium nova V445 Puppis (Iijima and Nakanishi, 2008).

As was mentioned above, aluminum has high cosmic abundance and astrophysical significance in the Universe. Its spectral lines are frequently present in stellar spectra (Smiljanic *et al.*, 2016, Carretta *et al.*, 2018, Smith, 1993). Aluminum abundances in giants and dwarfs were determined (Smiljanic *et al.*, 2016) and their implications on stellar and galactic chemical evolution were investigated. Aluminum abundances for red giant branch (RGB) stars in NGC 2808 were derived by Carretta *et al.* (2018). Smith (1993) reports atmospheric abundance of aluminum for a sample of normal, superficially normal and HgMn-type main-sequence late-B stars.

Similar as in the case of lutetium, the nucleosynthesis of zinc is not well understood (Barbuy*et al.*, 2015). To be investigated, abundances for various galactic stars are needed, as is underlined in Sneden*et al.* (1988). Zinc is also significant for: i) examination of star formation rate; ii) chemical enrichment of the Galactic bulge (Barbuy *et al.*, 2015, Da Silveira *et al.*, 2018 and references therein); iii) chemical evolution of the Universe at high redshifts, by studying abundances in damped Lyman-a systems (DLAs) (Pettini *et al.*, 1999, Rafelski *et al.*, 2012, 2014). For abundance determination, various atomic data for zinc, including Stark broadening parameters are of interest. Spectral lines of Zn III are observed in spectra of several O type stars, pulsators, etc. Lehmann*et al.* (2010) report observations of Zn III line in θ1 Ori C (O type star) from multiple stellar system Theta Orionis. Zinc abundance determination is published in Dorsch *et al.* (2019) for hot subdwarf stars of O type (sdO) HZ 44 and HD 127493 using spectral lines of Zn III an Zn IV. Zinc abundances were obtained for intermediate He-sdOB star: the pulsators Feige 46 and LS IV14^o116 where strong Zn III spectral lines were applied. Abundance determination of zinc needs Stark broadening parameters (Sneden *et al.*, 1988, Barbuy *et al.*, 2015). There are no experimental Stark broadening results in the literature where calculations for a few spectral lines were available, only.

All these astrophysical goals need reliable Stark widths and shifts that was the reason for our Stark broadening calculations. These data enter chemical abundance determination, modelling of radiative transfer and stellar atmospheres, analysis and synthesis of stellar spectra.

2. METHOD

The modified semiempirical method (Dimitrijević and Konjević, 1980) is developed to calculate the full width at half intensity maximum (FWHM) of an isolated spectral line, emitted or absorbed by a non-hydrogenic ion perturbed by surrounding electrons. When important spectroscopic data for non-hydrogenic ions are missed in the literature, and there is no enough data for more sophysticated methods, the modified semiempirical method could be used, what is its major advantage. According to the MSE the Stark width is given by the expression:

$$W_{MSE} = N\frac{8\pi}{3}\frac{\hbar^2}{m^2}\Big(\frac{2m}{\pi kT}\Big)^{1/2}\frac{\pi}{\sqrt{3}}\frac{\lambda^2}{2\pi c}\times$$

$$\times\{\sum_{\ell_i\pm1}\sum_{L_{i'}J_{i'}}\vec{R}^2[n_i\ell_iL_iJ_i,n_i(\ell_i\pm1)L_{i'}J_{i'}]\widetilde{g}(x_{\ell_i,\ell_i\pm1})+$$

$$+\sum_{\ell_f \pm 1}\sum_{L_{f'}J_{f'}} \vec{R}^2[n_f\ell_f L_f J_f, n_f(\ell_f \pm 1)L_{f'}J_{f'}]\widetilde{g}(x_{\ell_f,\ell_f\pm 1})+$$

$$+\big(\sum_{i'}\vec{R}^2_{ii'}\big)_{\Delta n\neq 0}g(x_{n_i,n_i+1})+\big(\sum_{f'}\vec{R}^2_{ff'}\big)_{\Delta n\neq 0}g(x_{n_f,n_f+1})\}. \quad (1)$$

where i and f indicate initial and final energy level of the studied transition. The square of the matrix element is given by expression:

$$\vec{R}^2[n_k\ell_k L_k J_k, n_k(\ell_k \pm 1)L'_k J'_k] =$$

$$\frac{\ell_>}{2J_k+1}Q[\ell_k L_k, (\ell_k \pm 1)L'_k]Q(J_k, J'_k)[R^{n^*_k(\ell_k\pm 1)}_{n^*_k\ell_k}]^2. \quad (2)$$

The quantity $x_{\ell_k,\ell_{k'}}$ in Eq. (1) denotes the ratio:

$$x_{\ell_k,\ell_{k'}} = \frac{E}{\Delta E_{\ell_k,\ell_{k'}}}, \quad k = i, f. \quad (3)$$

For $\Delta n \neq 0$, the energy difference between energy levels with n_k and n_k+1, is represented by the following equation:

$$\Delta E_{n_k,n_k+1} = 2Z^2 E_H/n^{*3}_k, \quad (4)$$

where $n^*_k = [E_H Z^2/(E_{ion} - E_k)]^{1/2}$ is the effective principal quantum number, N the electron density, T the temperature, Z the residual ionic charge, E_{ion} the appropriate spectral series limit, $Q(\ell L, \ell' L')$, multiplet factor and $Q(J, J')$ line factor (Shore and Menzel, 1965). Gaunt factors are denoted as $g(x)$ (Griem, 1968, 1974) and $\widetilde{g}(x)$ (Dimitrijević and Konjević, 1980). Radial integrals $[R^{n^*_k\ell_k\pm 1}_{n^*_k\ell_k}]$ are calculated by applying the Coulomb approximation (Bates and Damgaard, 1949), tables in Oertel and Shomo (1968) and the method of Van Regemorter *et al.* (1979), in cases when for higher atomic energy levels the tables in Oertel and Shomo (1968) are not applicable.

3. RESULTS

In order to demonstrate the importance of Stark broadening in astrophysics, particularly in the modelling of stellar atmospheres, we examined in Dimitrijević *et al.* (2024), by using the obtained results, three cases of plasma and surface gravity conditions. Stark broadening (full width at half intensity maximum) of singly ionized palladium spectral line $(^1G)5s^2G$ - $(^1G)5p^2H^o$, λ = 2486.36 Å is calculated versus Rosseland optical depth. It is compared with Doppler broadening for models of stellar atmosphere of: an A type star (T_{eff} = 8500 K and log g = 4.5); a DB white dwarf with an effective temperature of 25,000 K and log g = 8 and a DO white dwarf (T_{eff} = 60,000 K and log g = 8). Model of Kurucz (1979) is applied for an A type star and model of Wesemael (1981) for atmospheres of hot and high-gravity stars. Our results for Stark and Doppler dependance on the Rosseland optical depth outline the significance of Stark broadening mechanism of Pd II spectral lines in the atmospheres of hotter stars, DB and DO white dwarfs. It is the major broadening mechanism in these environments for the corresponding plasma conditions. This is valid even

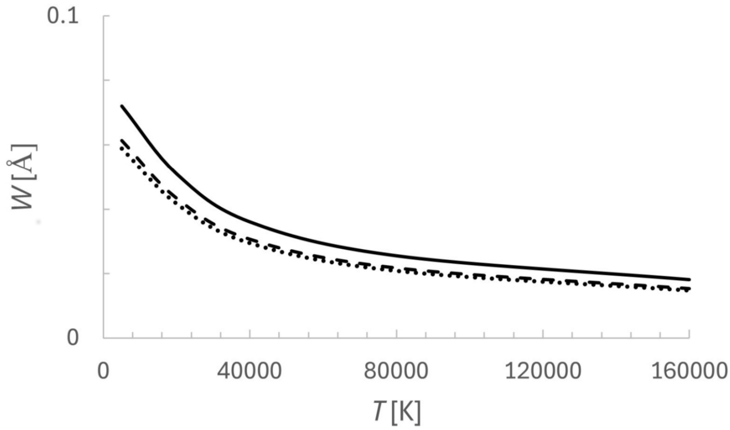

Figure 1: Temperature dependence of the Stark width for spectral lines from multiplet Pd II 5s^{4}F - 5p^{4}D^o for an electron density of 10^{17} cm^{-3}: 5s^{4}F$_{5/2}$ - 5p^{4}D$^o_{3/2}$ - dot line; 5s^{4}F$_{7/2}$ - 5p^{2}4D$^o_{7/2}$ - dash line; 5s^{4}F$_{5/2}$ - 5p^{4}D$^o_{7/2}$ - solid line.

for DO dwarf atmosphere with T_{eff} = 60,000 K, in spite of the increase of Doppler broadening with temperature. Stark widths are larger than Doppler ones, more than an order of magnitude for all examined optical depths, in this case. In the case of the considered A-type star atmosphere, Stark broadening is negligible within the whole range of examined optical depths. We found the same result in our recent studies of 19 spectral lines in the spectrum of singly charged lutetium ion (Dimitrijević *et al.*, 2021), 13 Si II multiplets (Dimitrijević *et al.*, 2023), 23 Al IV transitions (Dimitrijević and Christova, 2023), and 24 Zn III multiplets (Dimitrijević and Christova, 2022). However, in the case of A-type stars, there are spectral lines where Stark broadening is not negligible. An example are the Sn II lines, considered in Dimitrijević *et al.* (2024).

Additionally, we used the obtained results to show the Stark width dependence on the temperature. In Figure 1, results for three spectral lines from Pd II 5s^{4}F - 5p^{4}D^o multiplet is shown. The results for three lines (nine lines belong to this multiplet) include the largest and the smallest width within the considered multiplet. We see that the difference is noticeable. For example, for T = 20,000 K, the width of the highest curve is 22% larger than the width of the lowest one. If expressed in angular frequency units, the difference is 8.6%, which is much smaller. We note that Wise and Konjević (1982), on the basis of analysis of existing experimental Stark widths within multiplets, supermultiplets and transition arrays, concluded that the diferences between linewidths within a multiplet, if expressed in angular frequency units, are a few per cent, within a supermultiplet about 30% and in transition array within a range of about 40%. In the considered case the difference is larger than predicted by Wiese and Konjević (1982). This is because the difference between the highest and lowest energy level in the term 5p^{4}D^o is 5931.341 cm^{-1} and this value is not much smaller in comparison with the distance to the term 5s^{4}F. The temperature dependence of the Stark width for spectral lines from Pd II multiplets 5s^{2}F - 5p^{2}G^o and 5s^{4}F - 5p^{4}G^o is similar.

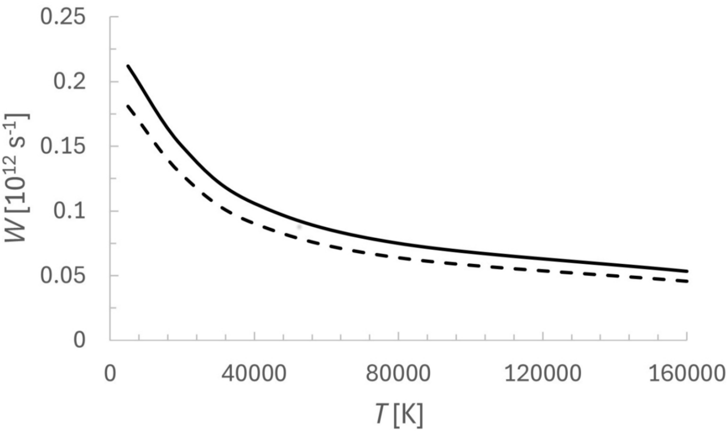

Figure 2: Variation of the Stark width in angular frequency units versus temperature for a supermultiplet Pd II $5s^2F$ - $5p^2L^o$ (L = G, F) for an electron density of 10^{17} cm^{-3}: $5s^2F_{5/2}$ - $5p^2F^o_{7/2}$ - dash line; $5s^2F_{3/2}$ - $5p^4D^o_{3/2}$ - solid line.

To illustrate the T-dependence of Stark broadening within a supermultiplet, we include such results in Figure 2 for the case of $5s^2F$ - $5p^2L^o$ (L = G, F) supermultiplet. The smallest and the largest width within this supermultiplet ($5s^2F_{7/2}$ - $5p^2F^o_{7/2}$, λ = 2715.7 Å and $5s^2F_{7/2}$ - $5p^2G^o_{9/2}$, λ = 2855.4 Å) are given in angular frequency units. The largest width is 31% larger than the smallest one when it is expressed in Ångströms and 17% in angular frequency units, which is well within the limits predicted by Wiese and Konjević (1982).

4. SUMMARY

Temperature dependence of calculated Stark widths belonging to Pd II spectral lines within a multiplet and supermultiplet is presented in this work. Possible applications of the results for astrophysical purposes are proposed. The significance of Stark broadening of Pd II spectral lines, in stellar atmospheres of hot stars from spectral type DB and DO white dwarfs is demonstrated. The modelling of their atmospheres according to Kurucz (1979) and Wesemael (1981) outlines that the principal broadening mechanism of singly charged palladium spectral lines for the corresponding conditions is Stark broadening. For A-type star atmospheres, the model of Kuruz (1979) indicates weak and negligeable influence of Stark broadening on the considered Pd II spectral lines. Discussion of Stark/Doppler broadening ratio behavior as a funcion of the optical depth is performed for previously published results for Lu II, Al IV, Zn III and Sn II spectral lines. The conclusion is that Stark broadening is very mporant for DB and DO white dwarf amospheres and that there are cases, as for example Sn II lines, where it is not negligible for A-type stars,

Stark broadening data of Pd II spectral lines could be applied for different goals in astrophysics, as abundance determination of palladium, analysis and synthesis of singly ionized palladium lines in stellar spectra, opacity calculations, spectroscopic diagnostics of laboratory and industrial plasmas, but also to investigate palladium plasma created by generation of high-order harmonics of laser pulses.

Acknowledgements

This work has been supported with a STSM visit grant E-COST-GRANT-CA18222-5308184a for M.S.D. within the framework of COST Action CA18222 Attosecond Chemistry - ATTOCHEM. We are kindly thankful to the Local organizing comity of the XIV Serbian Bulgarian Astronomical Conference to support our participation.

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