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THE EQUATION BETWEEN 3-BODY MEAN MOTION RESONANCES AND YARKOVSKY DRIFTS ON ECCENTRICITIES IN THE RANGE (0.1, 0.2)

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Abstract. We studied the motion of asteroids across the 3-body mean motion resonances (MMRs) with Jupiter and Saturn and with the Yarkovsky drift in the semimajor axis of the asteroids. The research was conducted using numerical integrations performed using the Orbit9 integrator with 72,000 test asteroids. We calculated time delays, $\mathrm{d}tr$, caused by the six 3-body MMRs on the mobility of test asteroids with 10 positive and 10 negative Yarkovsky drifts, which are reliable for Main Belt asteroids. Our final results considered only test asteroids that successfully crossed over the MMRs without close approaches to the planets. We devised equations that approximately describe the functional relation between the average time $\langle \mathrm{d}tr \rangle$ spent in the resonance, the strength of the resonance SR, and the semimajor axis drifts $\mathrm{d}a/\mathrm{d}t$ (positive and negative) with the orbital eccentricities of asteroids in the range $(0.1, 0.2)$. Comparing the values of $\langle \mathrm{d}tr \rangle$ obtained from the numerical integrations and from the derived functional relations, we analysed average values of $\langle \mathrm{d}tr \rangle$ in all 3-body MMRs for every $\mathrm{d}a/\mathrm{d}t$. The main conclusion is that the analytical and numerical estimates of the average time $\langle \mathrm{d}tr \rangle$ are in very good agreement, for both positive and negative $\mathrm{d}a/\mathrm{d}t$. Finally, this study shows that the functional relation we obtain for 3-body MMRs for orbital eccentricities of asteroids in the range $(0.1, 0.2)$ is analogous to that previously obtained for orbital eccentricities of asteroids in the range $(0, 0.1)$ in Milić Žitnik 2021.

1. INTRODUCTION

In our Solar system, a very accurate description of the orbital motion of small bodies does not exist, due to the complexity of gravitational and non-gravitational forces that influence their movement.

In the dynamics of asteroids, a very important mechanism are orbital resonances (Gallardo 2019). Especially, mean motion resonances (MMRs) modify the orbits of asteroids (Morbidelli & Moons 1995, Gladman et al. 1997). "The three-body MMRs seem to be the main actors structuring the dynamics in the Main Belt, because of their surprising overdensity in comparing to two-body MMRs" (Nesvorný & Morbidelli 1998). Moreover, Nesvorný & Morbidelli stated that the concept of three-body MMRs is important for explaining the chaotic behaviour of many asteroids. Further, Smirnov & Shevchenko (2013) located thousands of asteroids in three-body MMRs with Jupiter and Saturn in numerical integrations. One of their conclusions is that in three-body MMRs there are more asteroids than in two-body MMRs, which additionally emphasizes the importance of three-body MMRs.

In this study, I describe the dynamics of three-body MMRs (involving an asteroid, Jupiter and Saturn), taking a different dynamical approach than presented in the previously cited studies. I wanted to explore how much time, on average, an asteroid, influenced by the Yarkovsky effect, spent in three-body resonances. Yarkovsky force is an inevitable force in calculations of the orbital motions of an asteroid (Rubincam 1995, 1998; Farinella & Vokrouhlický 1999). The change of an asteroid's drift speed in the semimajor axis when it is crossing over the MMR is a result of the composition of secular drift in the semimajor axis (due to the Yarkovsky force) and its periodic oscillations (caused by an MMR). The result of this interaction is still enigmatic, so it is important to study this in future research. Another important non-gravitational force that affects the motion of asteroids is the well-known Yarkovsky-O'Keefe-Radzievsky-Paddack effect (YORP, Rubincam 2000). In our searches (Milić Žitnik & Novaković 2015, 2016; Milić Žitnik 2016, 2018, 2019, 2020, 2021), we did not take into account this non-gravitational effect. This research is a natural continuation of the paper Milić Žitnik (2021).

2. METHODS

The methods used to describe the interaction between a MMR and the Yarkovsky force were explained in great detail in Milić Žitnik & Novaković (2015, 2016) and Milić Žitnik (2019, 2020, 2021). A set of integrations of $72,000$ test asteroids ($6,000$ test asteroids for each of the 6 three-body resonances for both, positive and negative Yarkovsky drift) were obtained by measuring the semimajor axis drift delay inside the MMR, using the numerical integrator Orbit9[1] by Milani & Nobili (1988). For the orbit of test particles we took 10 values of $\mathrm{d}a/\mathrm{d}t$ from 4×10^{-5} to 2×10^{-3} au/Myr, both positive and negative equidistant values. In this study, we created 600 asteroids for every chosen value of semimajor axis drift speed with eccentricities in the interval (0.1, 0.2). We selected 6 isolated 3-body resonances with Jupiter and Saturn. We utilized a numerical method[2] described by Gallardo (2014) in order to estimate the strength of these MMRs. Finally, the method to get the average time that an asteroid spent in the resonance caused by the Yarkovsky force $\langle \mathrm{d}tr \rangle$, was used (Milić Žitnik & Novaković 2016).

3. RESULTS

Here are presented outputs of calculations of the influence of 3-body resonances on $\mathrm{d}a/\mathrm{d}t$ caused by the Yarkovsky force with asteroids' orbital eccentricities in the interval (0.1, 0.2). Only asteroids that successfully crossed over the 6 three-body resonances (asteroids that entered and exited from the MMR) without close encounters with planets are used. In Table 1 we presented the properties of the 6 selected 3-body resonances. The strength (SR) of all MMRs was computed with average values of $\langle e \rangle$ and of $\langle i \rangle$ for asteroids that successfully crossed over the resonances without close approaches to the planets for negative and positive Yarkovsky drifts and for $\omega = 60°$, where ω is the argument of pericenter. From the presented results, we can notice very similar values of SR in cases of negative and positive $\mathrm{d}a/\mathrm{d}t$.

[1]`http://adams.dm.unipi.it/orbfit/`

[2]`http://www.fisica.edu.uy/$\sim$gallardo/atlas/3bmmr.html`

Table 1: The properties of the 6 selected 3-body resonances. In the first column there are names of 3-body MMRs. In the second column their nominal semimajor axes are shown. The widths of the 6 selected 3-body resonances for $e = 0.2$ are in the third column, propagated by the numerical method with the Orbit9 (Milić Žitnik & Novaković 2016, Milić Žitnik 2018). The last two columns contain their strengths SR, in cases of negative and positive $\mathrm{d}a/\mathrm{d}t$, calculated with the numerical method given by Gallardo (2014).

MMR	a_{res} [au]	width($e = 0.2$) [au]	SR $\mathrm{d}a/\mathrm{d}t < 0$	SR $\mathrm{d}a/\mathrm{d}t > 0$
1:-3J:1S	2.75180	0.00350	0.00137283	0.00130213
2:-7J:3S	2.55896	0.00338	0.00099704	0.00086637
2:-7J:2S	2.44715	0.00200	0.00006268	0.00009325
1:-5J:6S	2.75761	0.00158	0.00000826	0.00000679
3:-6J:-1S	3.13787	0.00156	0.00001396	0.00003654
3:-8J:1S	2.79909	0.00092	0.00000877	0.00001229

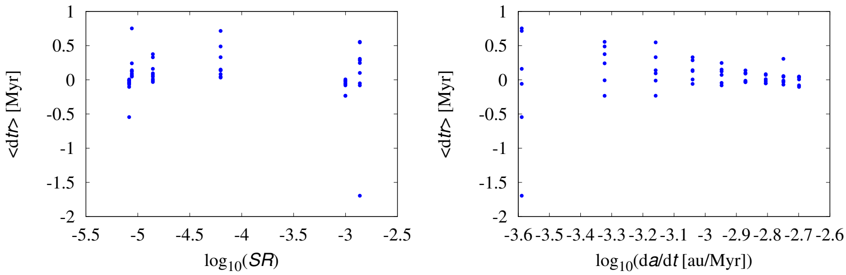

Figure 1: Relation between $\langle \mathrm{d}tr \rangle$ and $\log_{10}(SR)$ (left panel), and $\log_{10}(\mathrm{d}a/\mathrm{d}t)$ (right panel) with 9 absolute largest values of $\mathrm{d}a/\mathrm{d}t$, for negative Yarkovsky drifts.

The slowest speed -4×10^{-5} au/Myr (and also 4×10^{-5} au/Myr) was excepted from the further calculations, because asteroids with $\mathrm{d}a/\mathrm{d}t$ with absolute values less than 7×10^{-5} au/Myr typically cross over a mean motion resonance quickly (Milić Žitnik 2019). In Figure 1, the dependence of $\log_{10}(\langle \mathrm{d}tr \rangle)$ on $\log_{10}(SR)$ (left panel), and on $\log_{10}(\mathrm{d}a/\mathrm{d}t)$ (right panel) without -4×10^{-5} au/Myr is displayed. Obviously, $\langle \mathrm{d}tr \rangle$ in all resonances have negative or positive values. Despite the dispersion of outcomes, these results uncovered that a relation between $\langle \mathrm{d}tr \rangle$, SR and $\mathrm{d}a/\mathrm{d}t$ does exist. Furthermore, the 'log-log' scale was used because of the very small values of $\mathrm{d}a/\mathrm{d}t$ and SR. Conclusion is that absolute values of $\langle \mathrm{d}tr \rangle$ increase while SR is increasing. And, absolute values of $\langle \mathrm{d}tr \rangle$ increase while $\mathrm{d}a/\mathrm{d}t$ is increasing. I got very similar results for positive Yarkovsky drifts.

Considering the outcomes received in this research, I derived the equation that describes the correlation between $\langle \mathrm{d}tr \rangle$, SR and $\mathrm{d}a/\mathrm{d}t$ for e in the observed interval (0.1, 0.2):

$$\langle \mathrm{d}tr \rangle = (0.5 + \mathrm{d}a)\log_{10}(SR) + (\mathrm{d}b - 1.0)\log_{10}(\frac{\mathrm{d}a}{\mathrm{d}t}) + (\mathrm{d}c + 5.0), \tag{1}$$

where $\mathrm{d}a$, $\mathrm{d}b$ and $\mathrm{d}c$ are unknown parameters.

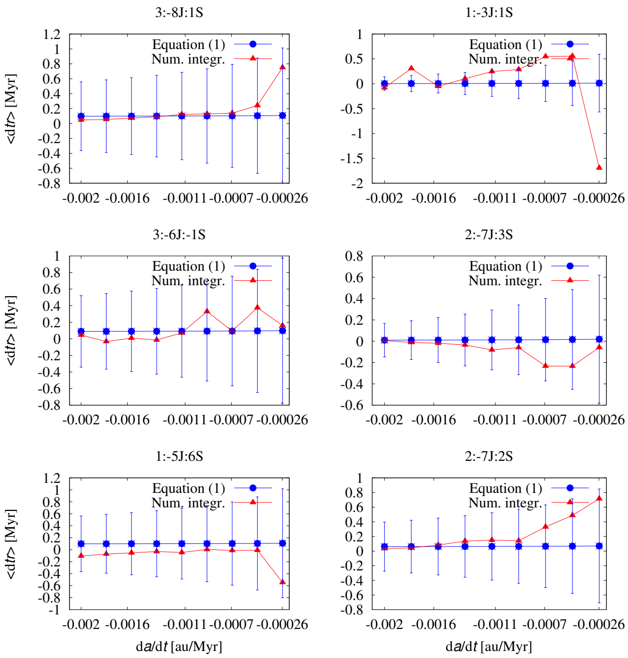

Figure 2: Values of $\langle \mathrm{d}tr \rangle$ obtained from Equation 1 in the 6 three-body resonances for negative $\mathrm{d}a/\mathrm{d}t$ for eccentricity in the interval (0.1, 0.2). Outcomes from Equation 1 are showed with 3σ interval error of $\langle \mathrm{d}tr \rangle$ from Equation 2.

Applying the least-squares method using the previous equation, we calculated unknown parameters and their standard errors, which represent the best equation between $\langle \mathrm{d}tr \rangle$, SR and $\mathrm{d}a/\mathrm{d}t$. For negative Yarkovsky drifts coefficients are $\mathrm{d}a = -0.543 \pm 0.049$, $\mathrm{d}b = 0.990 \pm 0.166$, $\mathrm{d}c = -5.147 \pm 0.545$ and for positive Yarkovsky drifts they are $\mathrm{d}a = -0.682 \pm 0.076$, $\mathrm{d}b = 0.729 \pm 0.244$, $\mathrm{d}c = -6.545 \pm 0.784$. Average time was expressed in Myr and the Yarkovsky drift was expressed in au/Myr.

In order to define the field of validity of Equation 1, I calculated errors of $\langle \mathrm{d}tr \rangle$ by differentiating the equation by required paremeters $\{\mathrm{d}a, \mathrm{d}b, \mathrm{d}c\}$ and got the following equation:

$$\sigma(\langle \mathrm{d}tr \rangle) = \sigma(\mathrm{d}a) \log_{10}(SR) + \sigma(\mathrm{d}b) \log_{10}(\frac{\mathrm{d}a}{\mathrm{d}t}) + \sigma(\mathrm{d}c). \tag{2}$$

Using Equation 2, I determined $3\sigma(\langle \mathrm{d}tr \rangle)$ errors for the 6 three-body MMRs and for negative $\mathrm{d}a/\mathrm{d}t$ in order to compare the analytical values of $\langle \mathrm{d}tr \rangle$ with its corresponding values from the numerical integrations. These results were shown in ($\langle \mathrm{d}tr \rangle$, $\mathrm{d}a/\mathrm{d}t$) plane (Figure 2). There is a good accordance in values $\langle \mathrm{d}tr \rangle \pm 3\sigma(\langle \mathrm{d}tr \rangle)$ calculated by Equation 1 and Equation 2 with the values $\langle \mathrm{d}tr \rangle$ from the numerical integrations, as expected.

In Figure 3 the same results in ($\langle \mathrm{d}tr \rangle$, $\log_{10}(SR)$) plane are presented. There is again a good accordance between $\langle \mathrm{d}tr \rangle \pm 3\sigma(\langle \mathrm{d}tr \rangle)$ obtained by Equation 1 and Equation 2, and on the other side corresponding $\langle \mathrm{d}tr \rangle$ from the numerical integrations. A good accordance exist for positive Yarkovsky drifts, also.

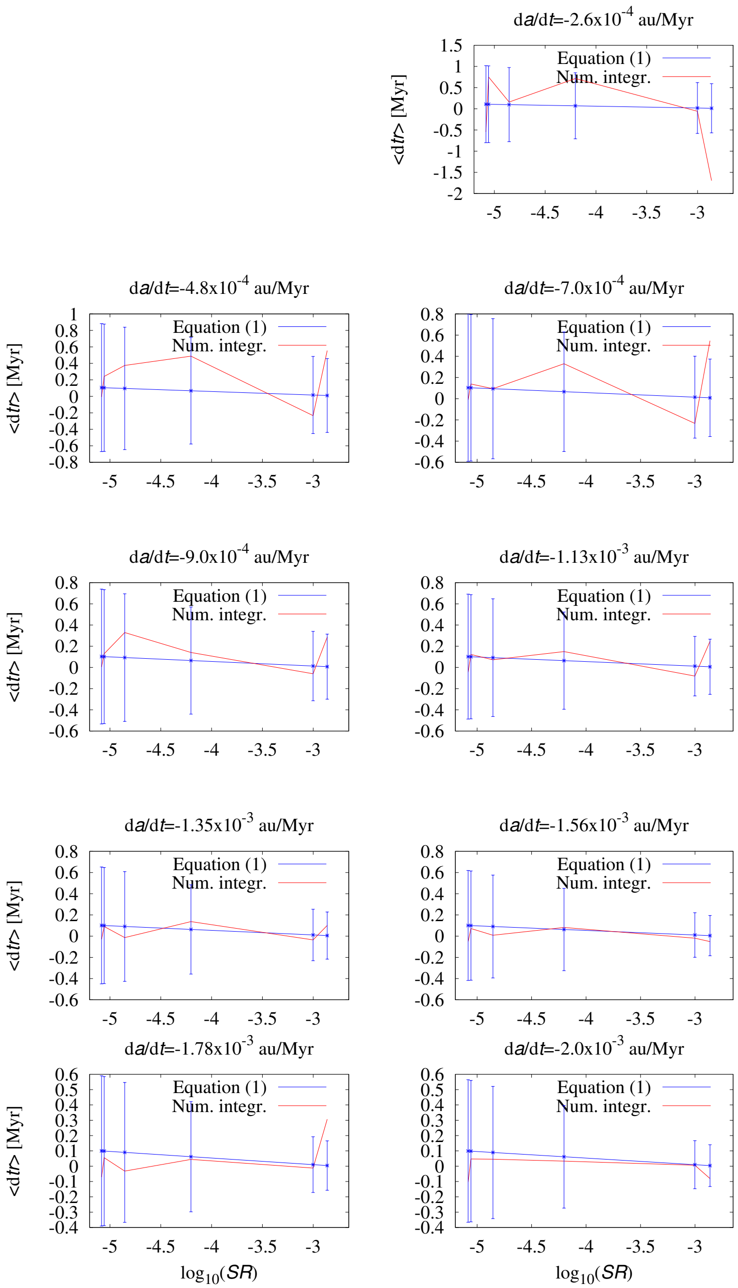

Figure 3: Difference of dependence $\langle \mathrm{d}tr \rangle$ on $\log_{10}(SR)$ in the 6 three-body resonances for negative $\mathrm{d}a/\mathrm{d}t$ between Equation 1 and the numerical integrations for eccentricity in the interval (0.1, 0.2). Outcomes from Equation 1 are showed with 3σ interval error of $\langle \mathrm{d}tr \rangle$ from Equation 2.

4. SUMMARY

I derived two equations that joined the time $\langle \mathrm{d}tr \rangle$ that an asteroid spent inside a three-body MMR, the strength of a resonance (SR) and the semi-major axis drift $\mathrm{d}a/\mathrm{d}t$ under the Yarkovsky effect, for eccentricities in the interval (0.1, 0.2). The equations enable quick propagation of the $\langle \mathrm{d}tr \rangle$ in a three-body resonance with known SR in the interval $[6.79 \times 10^{-6}, 1.37 \times 10^{-3}]$, with the negative and positive Yarkovsky drifts in the interval $[2.6 \times 10^{-4}, 2 \times 10^{-3}]$ au/Myr and with an asteroid's orbital eccentricity in the interval (0.1, 0.2).

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