



## ABOUT SOME NEAR-EARTH ORBITS

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**Abstract.** The population of near-Earth objects (NEOs) currently counts more than 30 000 known members. In general, NEOs have chaotic orbits, short lifetimes, and often an unidentified dynamical background. In this work, we discuss the discrepancy in the literature about the dynamical connection between the main belt asteroid 2 Pallas and the Apollo asteroid 3200 Phaethon, the parent body of the Geminides. We repeat the results on the Pallas-Phaethon link, using a still-developing Python platform running in REBOUND, and illustrate its sensitivity to some parameters of the numerical integration.

### 1. INTRODUCTION

Near Earth Objects (NEOs) are all comets and asteroids whose closest proximity to the Sun is below 1.3 AU. According to their orbits, NEOs are divided into three main groups: Amors with  $a > 1$  au and  $1.017 < q < 1.3$  au; Apollos having  $\geq 1$  au and  $q < 1.017$  au; and Athens with  $a < 1$  au and  $Q > 0.983$  au, where  $a$  is the semi-major axis,  $Q$  the aphelion and  $q$  the perihelion distance of the object.

Studying NEOs is of great scientific interest, firstly because they are one of the key issues in our understanding of the evolution of the Solar system, and possibly even life on Earth. NEOs are our closest cosmic neighbors, which makes them the most attractive targets for space missions, thus indirectly contributing to scientific and technological progress. The identification of potentially hazardous NEOs and the development of defense strategies is another important aspect of such studies.

In terms of the age of the Solar system, NEOs have a short dynamical life, less than 10 Myrs (Gladman et al 1997). Still, the current population of NEOs counts more than 30.000 known members and probably even more undiscovered ones, meaning that there is a dynamical mechanism that constantly brings new NEO members (otherwise, they would have been already cleared away in the 4.6 billion years-long history of the Solar system).

Several studies (Morbidelli and Gladman 1998, Bottke et al 2000, 2002, Granvik et al 2017, 2018, Nesvorný et al 2023) identified certain resonances as the main drivers from different parts of the Solar system to the NEO region (for more details, we propose the given literature). In this work, we will give a brief discussion of only one such dynamical path, acting between the large asteroid 2 Pallas located in the outer main belt, and the 5 km-sized Apollo asteroid 3200 Phaethon. This connection has already been studied in previous works (de Leon et al 2010, Todorović 2017, 2018, Kováčová et al 2022, MacLennan et al 2022), but since these investigations showed diverse and even contradictory results, we found it necessary to further discuss the Pallas - Phaeton relation.

## 2. THE POSSIBLE ORIGIN OF THE ASTEROID 3200 PHAETHON

The asteroid (1983 TB) 3200 Phaethon was discovered in 1983 by Green and Kowal. It turned out that 1983 TB was a debutante in many respects: it was the first asteroid discovered by a space telescope (IRAS), the first observed active asteroid, the first observed asteroid giving meteorite showers, the Geminides. At the time it was found, no other body had come this close to the Sun (its perihelion distance is  $q=0.139$  au), which inspired its name - Phaethon. Today, 40 years after its discovery, and more than 1000 papers discussing 3200 Phaethon (according to the SAO/NASA Astrophysics Data System search), it is still debated where it came from and how to explain the mechanism of its activity.

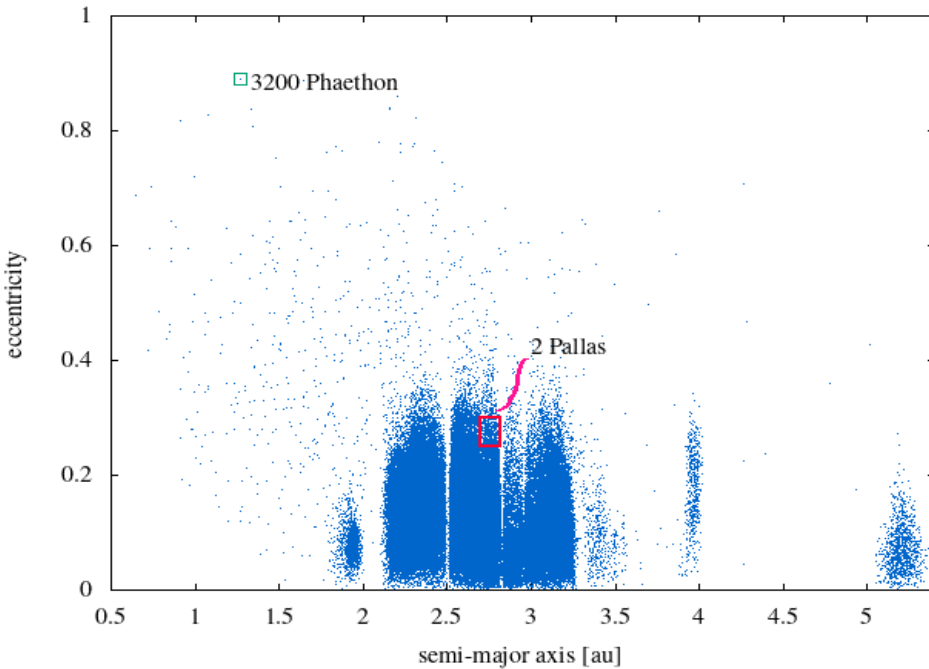


Figure 1: This plot shows the first 120 000 numbered asteroids (marked with blue dots) in the semi-major axis - eccentricity plane for  $[a, e] = [0.5, 5.4] \times [0, 1]$ . The location of the asteroid 3200 Phaethon at (1.27, 0.89) and the approximate location of the asteroid family 2 Pallas  $[2.7, 2.82] \times [0.25, 0.3]$  are marked with green and pink boxes respectively. The gap in the distribution of asteroids on the right border of the pink box represents the 5:2 mean motion resonance with Jupiter, which is recognized as one of the potential drivers to the current location of Phaethon. Our study is based on the assumption that the region framed with the pink box is the source region of the asteroid 3200 Phaethon and thereby the Geminides.

The possibility that 3200 Phaethon arrived from the asteroid family 2 Pallas was first proposed by de Leon et al 2010. The locations of these two asteroids in the semi-major axis - eccentricity plane are given in Fig.1, inside of the two boxes. The blue

dots are the first 120.000 numbered asteroids from the AstDys database<sup>1</sup>, where the gaps in their distribution represent the strongest mean motion resonances (MMRs). The right border of the pink box in which the asteroid 2 Pallas and the remaining members of its family are located, leans on the gap corresponding to the 5:2 MMR with Jupiter. Phaethon is obviously in another part of the Solar system on a highly eccentric and inclined orbit (the inclination is not visible in Fig.1).

Although mutually distant, authors in de Leon 2010 showed that Phaethon and Pallas have spectral similarities (both asteroids belong to the uncommon blue B-type), and may be dynamically connected via two MMRs with Jupiter, 8:3 and 5:2, with a probability of 2% in 100 Myrs. Later, Todorović confirmed this connection with a somewhat higher probability of about 8% in 5 Myrs (in Todorović 2017), and even 46% in 5 Myrs (in Todorović 2018). The significant variation in the outcomes is primarily attributed to the selection criteria for test particles (TPs) used in simulating orbital evolution. The selection of initial orbital parameters also largely influences the result (for more details see the above-mentioned articles). On the other hand, MacLennan et al 2021 and Kováčová et al 2022 used other methodologies and observed a low or zero possibility for Pallas-Phaethon dynamical connection. We will discuss these differences in more detail in a forthcoming paper, but here we will observe only the variations of the result within one methodology. We use the same method for searching the Pallas-Phaethon link as in Todorović 2018, but instead of making the calculations in Orbit9<sup>2</sup>, we repeat the integrations in REBOUND for different integration parameters. The calculations are done in a new platform relying on the REBOUND N-body integrator. A brief description is given below.

### 3. PLATFORM FOR DYNAMICAL MAPPING OF THE SOLAR SYSTEM

We present the still-developing platform designed to facilitate dynamical mapping of the Solar System using simple parameter configuration. The platform is written in Python and uses a REBOUND N-body integrator (Rein & Liu 2012) for calculating the motion of particles under the influence of gravity. Ephemerids of the objects in the Solar system are taken from the Jet Propulsion Laboratory (JPL) Horizons system. The platform's current functionalities include:

- i)* Generating ephemerids of chosen asteroids and planets from the JPL's Horizons system for a given epoch.
- ii)* Creating a dense grid of initial test objects for any pair of Keplerian elements in a given domain where the remaining Keplerian elements for all test objects will be either arbitrarily chosen or set to match the orbital plane of a specific asteroid or planet.
- iii)* Option to select planets (perturbers) to be included in the simulation.
- iv)* Creating stability maps using chaos indicator MEGNO (Mean Exponential Growth of Nearby Orbits; Cincotta & Simó 2000).

<sup>1</sup><https://newton.spacedys.com/astdys/index.php?pc=0>

<sup>2</sup><http://adams.dm.unipi.it/~orbmain/orbfit/>

- v) Identifying chaotic particles within a specified domain from stability maps and tracking their orbital evolution for a given period.
- vi) Choosing between different types of integrators in REBOUND, including symplectic integrators like WHFast (Wisdom-Holman integrator; Rein & Tamayo 2015), non-symplectic integrators like IAS15 (Integrator with Adaptive Step-size control, 15th order; Rein & Spiegel 2015), and hybrid symplectic integrators like MERCURIUS (Rein et al. 2019).
- vii) Monitoring if particles experience close encounters with planets, collide with planets, enter the vicinity of designated asteroids, or if particles classify as NEOs.
- viii) Detecting MMRs within the Solar system using the FAST method (Forgács-Dajka et al. 2018).
- ix) Calculating encounter manifolds of the planets with the careful selection of the initial conditions.
- x) Saving all simulation results in the files for further analysis with full control over how and what will be saved.
- xi) Plotting resonant angles.

Numerous additional tools and functionalities remain to be developed and integrated into the platform, such as creating stability maps for Fast Lyapunov Indicator (FLI) (Froeschlé et al 1997, 2000), identifying MMRs automatically, speeding up calculations (using GPU/CUDA), creating interactive stability maps, incorporating advanced plotting (using GNU PLOT), among others. In the following, we show some examples of the platform, in the context of studying the Pallas-Phaethon connection.

### 3. 1. THE STABILITY MAP

The first example will demonstrate the calculation of the map of the 5:2 MMR with Jupiter. More precisely, we map only one narrow region of the resonance where we estimated that the Pallas family members may have been injected (i.e. we reproduce Fig. 2 in Todorović 2018). For this purpose, we established a densely distributed grid of initial test objects within the  $(a, e)$  domain for the specified epoch. We generated a stability map for the designated time using the chaos indicator MEGNO and the symplectic integrator WHFast. Fig. 2 illustrates the resulting map with regions featuring chaotic particles (colored yellow), represented by higher MEGNO values, and relatively stable sections with lower MEGNOs (colored blue). The map reveals the main chaotic structures observed in Fig. 2. from Todorović, although lacking some of its clarity and content.

The platform uses the multiprocessing module to run the simulations in parallel to gain speed and efficiency. Generating a map of 1 million test objects including all planets and integrating over 5000 years with a time step of 1 year, takes approximately 2.5 hours on Ubuntu 22.04 LTS, AMD® Ryzen 9, 128 GB RAM. Let us note that the calculation of the map in Todorović 2018 in the Orbit9 software, which employed 250 000 TPs over the same period of 5000 years took about 7 hours. Calculating the same map on a cluster takes considerably less time.

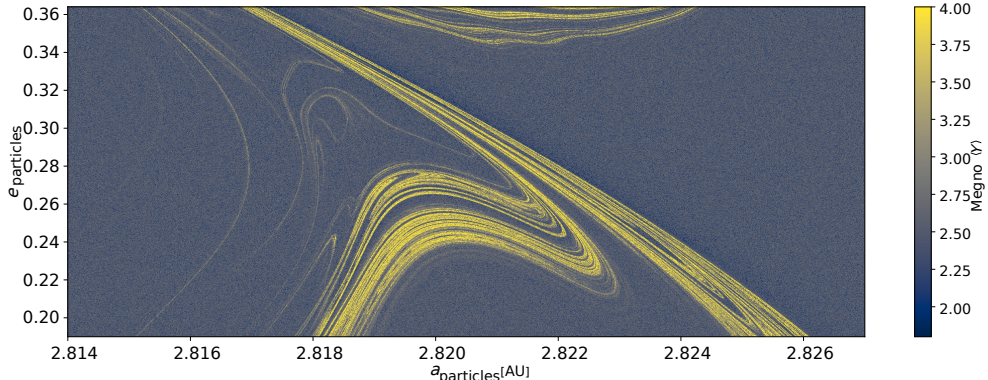


Figure 2: One part of the 5:2 mean motion resonance with Jupiter calculated in the orbital plane of the asteroid 2 Pallas. Using the new platform, we reconstructed the same region as in Todorović 2018, but for a higher resolution and significantly shorter calculation time. Chaotic structures are colored yellow, while stable ones are blue. The map reveals the same structures as in Fig. 2. from Todorović, but with somewhat less clarity and content.

### 3. 2. IDENTIFYING RESONANCES.

Let us use the example of the 5:2J MMR to give a short reminder on how to estimate the location of a given resonance. The 5:2 mean motion resonance with Jupiter appears when an asteroid makes five revolutions around the Sun, while Jupiter makes two i.e. when  $5T_{ast} = 2T_{Jup}$ , where  $T_{ast}$  and  $T_{Jup}$  are the orbital periods of the asteroid and Jupiter, respectively. According to the third Kepler law we have that  $(a_{ast}/a_{Jup})^3 = (T_{ast}/T_{Jup})^2$ , with  $a_{ast}$  and  $a_{Jup}$  being the semi-major axis of the asteroid and Jupiter ( $a_{Jup} \sim 5.2$  au). The unknown variable is the location of the resonance  $a_{ast} = a_{Jup}(T_{ast}/T_{Jup})^{2/3}$ , which is approximately  $a_{ast} \sim 2.82$  au for the commensurability 5:2.

In the other example demonstrating the platform abilities, we will show how to identify a 2-body MMR, using the method described in Forgács-Dajka et al. 2018. The test asteroid was taken from the stable part of the resonance in Fig. 2. In Fig. 3 we display the plot  $\lambda_{Jup} - \lambda_{TP}$  versus  $M_{TP}$  (panel a), where  $\lambda_{Jup}$  is the mean longitude of Jupiter, while  $\lambda_{TP}$  and  $M_{TP}$  are the mean longitude and mean anomaly of the test asteroid. The FAST method counts the number of intersecting stripes with the horizontal and vertical axes, which gives us  $q = 3$  and  $p + q = 5$  values for the particle in Fig. 3. Thus, the ratio of the mean motions is  $(p + q)/p = 5/2$ . The liberation of its resonant angle  $\theta_{TP}$  for 10000 years is given in panel b.

### 3. 3. PALLAS-PHAETHON CONNECTION FOR DIFFERENT INTEGRATION STEPS

In the last example, we will use the platform to verify how efficient is the 5:2J MMR in transporting bodies to the NEO region, i.e. to the neighborhood of the asteroid 3200 Phaethon. The initial conditions, the epoch, and the integration times are the same as in Todorović 2018. We take 1000 TPs from the chaotic structures in Fig. 2, integrate them for 5 million years and observe which ones arrived close to Phaethon. We repeat the same calculations for three different integration steps,  $dt = \{1, 0.1, 0.01\}$  years.

Table 1: The table shows the percentage, first arrival and median transportation times for 1000 particles taken from the 5:2J MMR in the region of the 2 Pallas family. Each particle is tracked for 5 Myrs. The three columns give three different results for different integration steps of  $dt = \{1, 0.1, 0.01\}$  years. The rate of arrived particles becomes about two times smaller for changing the step from  $dt = 1$  to  $dt = 0.01$ . The first arrival times become notably longer (from 314 370 yrs to 583 700yrs), while the smallest increase is observed for the median arrival times, they are all about 2 Myrs.

dt [yrs]	dt = 1	dt = 0.1	dt = 0.01
arrived [%]	15.70%	20.00%	7.40%
time of first arrival [yrs]	314.370	437.490	583.700
median time [yrs]	1.973.070	2.046.705	2.181.755

The percentage of arrived objects and their first and median arrival times to the neighbourhood of Phaethon are given in Table 1.

The Pallas-Phaethon dynamical link was observed for all three runs, with probabilities of 15.70% for  $dt = 1$  yr, a somewhat higher rate of 20.00% for  $dt = 0.1$  yr, and only 7.40% for the smallest integration step of  $dt = 0.01$  yr. It is a much lower percentage than the 46% observed in Todorović 2018 with Orbit9, but a higher rate than in Kováčová 2022 using REBOUND. First arrival times increase almost twice (from 314 370 to 583 700 yrs) by decreasing integration steps, while the median times are about 2 million years for all three runs, although showing a slight increase for smaller  $dt$ .

This same calculation was repeated for several different sets of 1000 test objects taken from the chaotic border of the resonance. They all gave very similar results. Chaotic orbits are by definition sensitive to initial conditions, but what may not be sufficiently addressed in the literature, is their sensitivity to integration parameters.

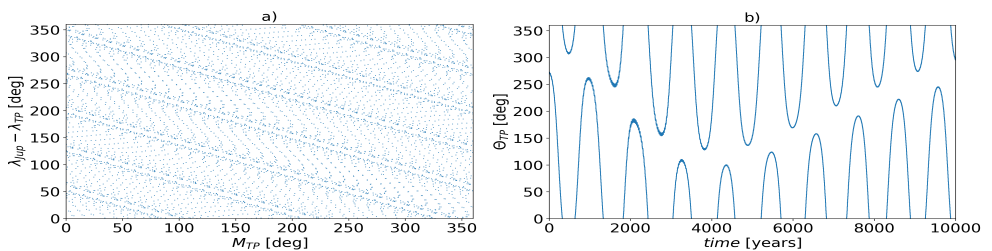


Figure 3: a) Test object that represents an asteroid in a 5:2 MMR with Jupiter. b) Corresponding resonant angle  $\theta_{TP} = 5\lambda_{Jup} - 2\lambda_{TP} - \varpi$  showing libration in 10 000 years.

#### 4. CONCLUSION

We discuss the discrepancies in the literature about dynamical pathways between the main belt asteroid 2 Pallas and the asteroid 3200 Phaethon in the NEO region, acting via the 5:2 mean motion resonance with Jupiter. We also present our ongoing work on

a platform, which made mapping and the selection of test objects for diffusion easier. Our example shows that results significantly differ for changing only the integration step, although the remaining parameters match. Therefore, it is not surprising that different methodologies of different authors give radically different results especially when it comes to the integration of chaotic orbits.

### Acknowledgements

This research was supported by the Ministry of Science, Technological Development and Innovation of the Republic of Serbia under contract number 451-03-66/2024-03/200002. The authors thank the organizers of the XX Serbian Astronomical Conference for their commitment, effort, and enthusiasm. N. T. is grateful for the opportunity to give an invited talk.

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