Publ. Astron. Obs. Belgrade No. 104 (2024), 159 - 167 *Progress Report*
https://doi.org/10.69646/aob104p159

DYNAMICS OF SPIRAL GALAXIES IN NONLINEAR REGIME - NONLINEAR SOLITARY WAVES IN ACCRETION DISK

MIROSLAVA VUKČEVIĆ, EDI BON and NATAŠA BON
Astronomical Observatory, Volgina 7, 11060 Belgrade, Serbia
E–mail: mvukcevic@aob.rs

Abstract. Theory of accretion disk is subject of several different theoretical models. Here, we list all important effects that can influence the accretion disk dynamics. For each effect, we propose nonlinear solution that is able to overcome linearly unstable structure. Such a stable, long lasting structure is useful in many aspects, but the most important one is possibility to compare to observed parameters relevant for the existence of the nonlinear solution.

1. INTRODUCTION

The nuclei of some galaxies are the subjects of large luminosity that are not produced by stellar mechanisms but rather by strong dynamical activity and high energy events. The energetic requirements are commonly interpreted as gravitational energy release by accretion mechanism onto massive black hole. Accreted gas has high angular momentum, so that, it forms an accretion disk. If the gas conserves angular momentum but is free to radiate energy, it will lose energy until it is on a circular orbit of radius $R_c = j^2/(GM)$, where j is the specific angular momentum of the gas, and M is the mass of the accreting compact object. The gas will only be able to move inward from this radius if it disposes of part of its angular momentum. In an accretion disk, angular momentum is transferred by viscous torques from the inner regions of the disk to the outer regions. The presence of rotation in the galactic nuclei suggests that accretion flow takes the form of the disk with a viscous dissipation. For the accretion onto black hole with particles following quasi Keplerian orbits the innermost stable orbit is $R_{ISCO} = 6R_g$ for nonrotating black holes, where $R_g = GM/c^2$ is gravitational radius. In more realistic, rotating case, one has to consider Kerr metric implying much lower stable orbits, below $R = 6R_g$ and all the way to the innermost stable orbit of $R_{ISCO} = R_g$. Accretion of the mater onto the massive object in the center should be followed by the outward transfer of the angular momentum. Several mechanism have been proposed in explaining this outward flow: enhanced turbulence in the hydrodynamic flows (Shakura & Sunyaev 1978), or by investigation of possible instabilities that can occur in the magnetohyadrodynamic flows (Velikhov 1959, Chandrasekhar 1960). Neither of these two mechanism resolve the angular momentum transfer satisfactory. The first one was unable to establish the origin and level of turbulence (Papaloizou & Lin 1995), while the second one faced the numerical issues: three dimensional numerical magnetohydrodynamic simulations achieved significant outward transport of angular momentum but in very local regime that could not be extended on the scale large enough (Balbus & Hawley 1998), and even more, predicted conductivity was so small leading to the negligible coupling between matter and the field. The rotational effects have been neglected, so that, we found that it would be of interest to try model the accretion disk (AD) including all three effects in order to scale them and to include nonlinear terms in order to check if possible to balance some of them.

The observed variability of the emitted light from the Type 1 AGN are detected in the continuum and broad emission line shapes. In particular, the broad emission line shape variability shows patterns that indicate behaviour which could be explained assuming the orbiting instability of the AD, in close proximity to the central SMBH (Sniegowska et al. 2020). Instabilities in the accretion disk could result with formation of spiral density wave (Smailagić & Bon 2015),

that would illuminate the BLR, resulting with changes of broad emission line shapes (Chakrabarti 1995). These effects could be traced in long term monitorings of spectral broad emission line variations (Gezari et al. 2007), and even in reverberation campaigns (Du & Wang 2023).

The Rossby wave instability of accretion disks with rotation but no magnetic filed effects has been reported in (Lovelace & Hohlfeld 1978). Close inspection of the local WKB dispersion relation for the unstable modes in the galactic disk shown an analogy with the instability regime of Rossby waves in planetary atmospheres (Marcus 1989). It has been found that those unstable modes are related to the gradient of the vorticity of the potential (Yecko 1995) on contrary to the usual Rossby waves derived for an incompressible fluid for which instability is caused by gradient of the fluid thickness.

In the linear theory of inverse vorticity it was assumed Keplerian motion, which means that the problem has not been treated self-consistently but rather it is imposed to assumed potential field. In this work the potential has not been assumed; the set of fluid equations is accompanied by Poisson's equation for the gravity potential.

An important property that distinguishes the inner from the outer region of galaxy is the relatively large angular velocity and large velocity dispersion. According to observed rotational velocity curve of a number of galaxies, inner region mostly rotates as a solid body as whole, but in smaller scale, inside a different regions of the bulge, fluid rotates differentialy. However, we investigate influence of both, solid body rotation and differential rotation, together with finite thermal pressure and self-gravity.

Motivation to investigate these particular regions in nonlinear regime rises because there are different structures observed and still there is no an unique explanation for them. One approach could be that spirals are created due to distortion or evolution of the localized soliton-vortex structure as it was observed in experiments carried for plasma vortex dynamics or number of computer simulations (Fine et al. 1991, Sommeria et al. 1989). The aim is to find out necessary condition for existence of both localized solutions within the geometry and physical conditions valid for the inner part of galaxy. It will be of particular interest to find out if it is possible to derive analytically soliton solution in the case of accretion disk since the soliton is thought to be able to dump the turbulence.

The type of the nonlinear equation depends on dispersion properties of the system and the type can be predicted from the analysis of linear dispersion relation. Linear analysis is necessary before derivation of the nonlinear equation in order to define stable parameter regime which includes necessary condition for the soliton existence.

There are different methods used to obtain nonlinear equation but all of them are based on perturbation process.

In accretion disks dynamics, at certain conditions, the weak self-gravity at certain distances can be important in forming vortices and spiral patterns (Lin & Pringle 1987). Such perturbations may give a bump in $L(r)$. The bump in $L(r)$ is crucial because it leads to trapping of the wave modes in a finite range of radii encompassing the corrotation radius. In this paper we propose localized stable nonlinear solution for Rossby waves in vortex shape. The stable nonlinear formation is possible even for the constant thickness of the layer. Further research regarding accretion disk dynamics could be referred to the magnetic field influence as it has been done in the Earth's ionosphere dynamics (Vukcevic & Popović 2020).

2. METHODS

The study of accretion flows is based on hydrodynamic models. The main problem is posed on the self-consistent solution of set of differential equations accompanied by Poisson equation when the self-gravity is incorporated. The density wave model consists of the transport equations of the mass density ρ and the momentum ρv, together with Poisson's equation that relates the density and the gravitational potential ϕ. The transport equations constitute a system of first-order hyperbolic partial differential equations. The dispersive property originates from the coupled Poisson's equation that is a second-order elliptic partial differential equation. The geometry of the system, therefore, influences the dispersion relation through Poisson's equation. The model of Lin and Shu assumes an infinitely thin disk and approximate Poisson's equation by

$$\frac{\partial \phi}{\partial r} \propto i\rho, \tag{1}$$

where in vertical direction has been assumed Dirac's delta function (Lin & Shu 1964) and potential ϕ and density ρ are two-dimensional.

In the core region, however, this approximation may not work, because the thickness is no longer negligible. New treatment of the Poisson's equation has been proposed that led to novel dispersive property of the system. It has been shown that geometry of the problem plays essential role through the Poisson's equation in dispersive properties of the system (Vukcevic 2019). Finite thickness of the disk is studied separately by several authors (Vandervoort 1970, Romeo 1992) implying that in contrast to the thin disk, the thick disc has to be hotter vertically than radially. Main concern of this work is to include the self-gravity and thickness effects via Poisson's equation, solving the dynamics of the system self-consistently together with continuity equation and equation of motion.

Let us first consider a model of the inner part of a galaxy with solid body rotation. Then, it is described by following set of equations

$$\begin{cases} \dfrac{\partial}{\partial t}\sigma + \{\phi, \sigma\} = 0, \\ \\ \sigma = -A\nabla_{\perp}^2\phi + B\phi, \end{cases} \tag{2}$$

where $\{\ ,\ \}$ is the Poisson bracket defined by $\{a,b\} = (\nabla_\perp b) \times (\nabla_\perp a) \cdot e_z$ for constant thickness of the system, while if thickness is not constant but rather radially dependent, it is described by

$$\begin{cases} \dfrac{\partial}{\partial t}\sigma + \{\phi, (\sigma_0 + \sigma)\} = 0, \\ \\ \sigma = -A\nabla_\perp^2 \phi + B\phi. \end{cases} \tag{3}$$

Here, the subscript $\perp$ indicates the components perpendicular to the direction of angular velocity e_z and ϵ is small parameter of the order of $(2\Omega_0)^{-1}\frac{d}{dt}$, where Ω_0 is the average angular velocity. Last assumption is consistent with the condition of existence of a drift wave and physically means that fluid inertia in the direction of the ambient rotation is negligible.

Vector type of the nonlinear term connected to the $[\nabla\phi \times \nabla]_z$ is responsible for the specific vortex-type solutions, namely Rossby waves. However, in order to obtain integrable nonlinear solution it is necessary to incorporate scalar type of nonlinearity discussed bellow.

We discuss the stationary waves assuming that $\phi = \phi(y - ut, x)$, where u is a constant wave group velocity along y. x and y are local coordinates along radial and azimuthal direction respectively. Then, Eq. (3) takes the form

$$\begin{array}{c} -2u\Omega_0 \frac{\partial}{\partial y}(B\phi - A\nabla^2\phi) - \frac{1}{2}B'\frac{\partial}{\partial y}\phi^2 + A(\nabla\phi \times \nabla)_z \nabla^2\phi + \\ +(\phi_0' B - \sigma_0')\frac{\partial}{\partial y}\phi - \phi_0' A \frac{\partial}{\partial y}\nabla^2\phi = 0, \end{array} \tag{4}$$

where $'$ denotes derivative with respect to x. There are two nonlinear terms: one is vector-type responsible for some turbulent motion, and the other is connected with the gradient of the thickness B'. Then, Eq. (4) has following soliton solution

$$\phi = \frac{2\lambda}{\nu}\psi(R), \tag{5}$$

where $R = \sqrt{\lambda}r$ is the dimensionless radius in the moving frame. Coefficients λ and ν reads as:

$$\lambda(x) = \frac{B}{A} - \frac{\sigma_0'/\sigma_0}{u/\Omega_0}, \tag{6}$$

$$\nu(x) = \frac{A}{2u/\Omega_0}(\lambda A' + \sigma_0''/\sigma_0), \tag{7}$$

where $''$ represents second derivative with respect to x, and ψ is the solution of the equation

$$\frac{1}{R}\frac{\partial}{\partial R}R\frac{\partial\psi}{\partial R} = \psi - \psi^2. \tag{8}$$

The Eq. (8) has approximate solution (Zakharov & Kuznetsov 1974)

$$\psi = 2.4(\cosh(\frac{3}{4}R))^{-\frac{4}{3}}. \tag{9}$$

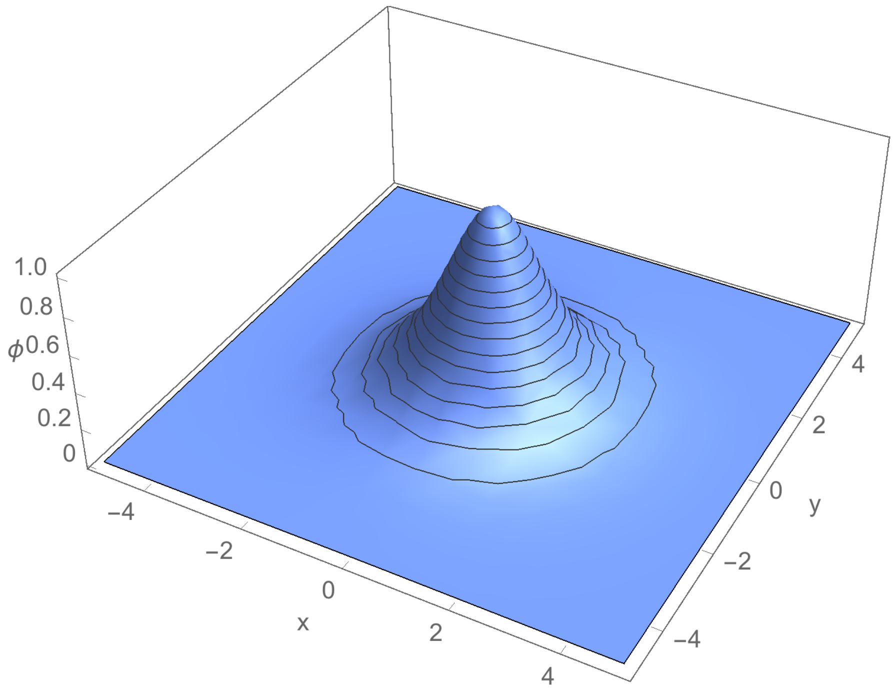

Figure 1: Symmetric soliton vortex obtained for the gravity potential in the inner part of the galaxy by Eq. (8). z axis is relative amplitude of the potential, while x and y are expressed in [light-days]. Both dimensions are dependent on the black hole mass; in this case black hole mass is 10^8 M$\odot$.

The potential has form of steady solitary vortices shown in Fig. 1, where R represents dimensionless distance to the center of the vortex. Vortex is traveling along y coordinate with u.

Large number of edge-on galaxies, show radial dependence of the thickness of the bulge and sharp gradient in vertical direction between bulge and galactic disk in all wavelenths. In the case of accretion disk, it has increasing thickness with respect to the distance from the SMBH (see Fig.4 in Shakura & Sunyaev (1973)). In our calculation this radial dependence results in A and B to not be constant any more, but rather r dependent. However, it has been shown that even for the $B = const.$ this particular soliton solution exists due to specific mutual influence of A and B on the parameters λ and ν (Vukcevic 2019).

The low-frequency perturbations compared to the angular frequency have been considered according to small parameter $(2\Omega_0)^{-1}\frac{d}{dt}$. This assumption is well satisfied for most of fluids in galactic case as the timescale for nonlinearity to occur and develop is larger than the average rotation period of the fluid. The geometry influence on dispersive properties that could be counterbalanced by nonlinear effects resulting in localized solitary-vortex has been analyzed. Although similar approach using so-called geostrophic approximation is widely applied on number of fluids such as planetary atmospheres and plasma waves (Hasegawa, Maclennan & Kodama 1979, Yecko 1995, Sommeria, Meyers & Swinney 1989), and in some of them possible soliton solution has been discussed (Petviashvili 1983), here we concentrate on the main contribution of the Poisson's equation that was in cited papers either simplified due to specific relation between quantities relevant for considered system, or neglected due to following different approach or geometry.

3. VORTEX SOLUTION: DIFFERENTIAL ROTATION AND MAGNETIC FIELD

It has been shown in previous section that when the effects of solid body rotation and finite velocity dispersion are significant it is possible to derive the solitary-vortex solution.

In that case, $\nabla \boldsymbol{v} \neq 0$, and consequently, involved parameters related to nonlinear and dispersive term will be more complex comparing to the uniform rotation case. In this section, we analyze necessary condition in order to derive solitary-vortex solution if possible, and eventual restriction on the rotation velocity related with potential equilibrium property. The type of the vortex depends on the sign of extremum value for density inhomogeneity in the equilibrium (cyclones by minimum, or anticyclone by maximum). The approximate solution will be elongated along the x due to dependence of f on x via $\Omega(x)$. In the case when angular velocity of the fluid cannot be treated as constant value (solid body rotation does not take place due to to viscous effects), nonlinear equation has the following form

$$\begin{gathered}\tfrac{1}{2}(\tfrac{1}{\Omega}B' - \tfrac{\Omega'}{\Omega^2}B)\tfrac{\partial \phi^2}{\partial y} + (\tfrac{1}{\Omega}\sigma_0' - B\Omega - \tfrac{1}{\Omega}B\phi_0' - \sigma_0\tfrac{\Omega'}{\Omega^2})\tfrac{\partial \phi}{\partial y} + \\ (\tfrac{1}{\Omega}\phi_0' A + \Omega A)\tfrac{\partial}{\partial y}\nabla_\perp^2\phi - \tfrac{1}{\Omega}A(\nabla\phi\times\nabla)_z\nabla_\perp^2\phi + (\tfrac{\Omega'}{\Omega^2}A - \tfrac{1}{\Omega}A')\nabla^2\phi\tfrac{\partial\phi}{\partial y} = 0.\end{gathered} \tag{10}$$

Last term in Eq. (10) can be neglected since it contains higher order nonlinearities. Dividing all terms by ΩA, one obtains

$$\frac{\partial}{\partial y}\nabla_\perp^2\phi - \frac{1}{\Omega^2}(\nabla\phi\times\nabla)_z\nabla_\perp^2\phi + \frac{1}{A\Omega^3}\frac{\partial\phi}{\partial y}(\sigma_0'\Omega + \Omega^3 B - \sigma_0\Omega') + \frac{1}{2A\Omega^3}(\Omega B' - \Omega' B)\frac{\partial\phi^2}{\partial y} = 0. \tag{11}$$

The approximate solution of Eq. (11) will be

$$\phi = \frac{\Lambda}{\Pi}f(R), \tag{12}$$

where f is

$$f = \cosh(\frac{3}{4}R(1+\Omega(x)\Lambda))^{-\frac{4}{3}}, \tag{13}$$

and Λ coincides with λ and Π coincides with ν.

The approximate solution will be elongated along the x due to dependence of f on x via $\Omega(x)$.

As far as the magnetic influence is concerned, that influence can be included via inclusion of current equation and electric dynamo filed equation.The system of equations is closed and it can be solved self-consistently by making a vector product of equation of motion and $\boldsymbol{e_z}$ giving

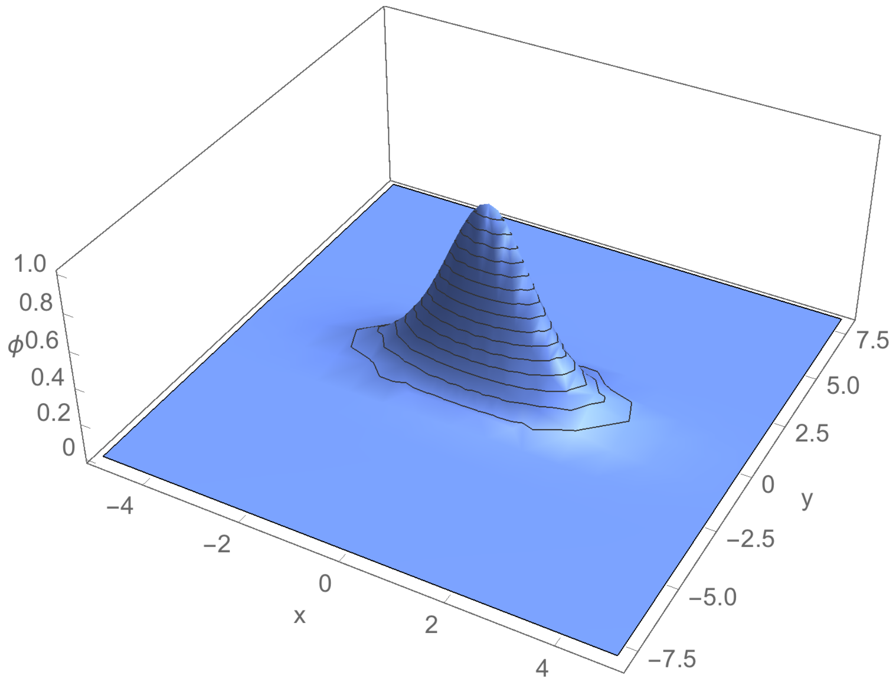

Figure 2: Solitary vortex shape of the dimensionless scalar potential elongated along the x axis due to differential rotation or/and magnetic effects.

$$\begin{aligned}(\tfrac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v}) \times \boldsymbol{e_z} + 2(\boldsymbol{\Omega}\times\boldsymbol{v})\times\boldsymbol{e_z} + \tfrac{1}{\rho}(\boldsymbol{j}\times\boldsymbol{B_0})\times\boldsymbol{e_z} \\ = (\nabla\phi + \tfrac{1}{\rho}\nabla p)\times\boldsymbol{e_z},\end{aligned} \tag{14}$$

where ρ and p are density and pressure of neutral gas respectively, influenced by charged particles electrons and ions, and B_0 is magnetic field induction and j is electric current.

Let us now investigate in details second and third term in the lefthand side of Eq. (14) denoting them as $f_R = |2(\boldsymbol{\Omega}\times\boldsymbol{v})\times\boldsymbol{e_z}|$ and $f_H = |\frac{1}{\rho}(\boldsymbol{j}\times\boldsymbol{B_0})\times\boldsymbol{e_z}|$. f_0 represents coupled rotational and magnetic filed contribution defined as $f_0 = f_R + f_H$, so that corresponding equation reads as:

$$\begin{aligned}-0.2uf_0\tfrac{\partial}{\partial y}(B\phi - A\nabla^2\phi) - \tfrac{1}{2}B'\tfrac{\partial}{\partial y}\phi^2 + A(\nabla\phi\times\nabla)_z\nabla^2\phi + \\ +(\phi_0'B - \sigma_0')\tfrac{\partial}{\partial y}\phi - \phi_0'A\tfrac{\partial}{\partial y}\nabla^2\phi = 0,\end{aligned} \tag{15}$$

with the following solution:

$$\psi = \cosh(\frac{3}{4}R(1+|f_0|\lambda))^{-\frac{4}{3}}, \tag{16}$$

where $|f_0|$ has to be derived using Eq. (14) and it depends on the ionization degree of the gas. Potential in this case is shown in Fig. 2 and it is elongated along the x axes due to coupled viscous and/or magnetic filed effects.

4. SUMMARY AND DISCUSSION

We have analyzed possible derivation of integrable nonlinear equation in the inner part of galaxy, particularly in the accretion disk. Accretion disk is complex region that can be treated in several different regimes according to observational data. It can be divide itself in several layers with respect to distance from the center where is supposed to be placed massive object. That layers differ in many properties such as density, rotation speed, thickness and temperature. Having in mind these four important parameters that characterize each part of the accretion disk, we discuss necessary condition for different type of nonlinear equation to be derived. Conditions are defined by specific parameters relation that needs to satisfy certain assumptions in order to explain observed structures. These localized solutions called solitons can be created at different regions of the disk depending on the type of rotation. The rotation type will also cause the shape and size of the structure. All these parameters are not independent and they are defined by the equilibrium functions. In particular, size of the soliton is defined by certain relation between gradient of gravitational potential and soliton speed, as well as by radial and vertical length scales. The nonlinear vortex is long lasting two-dimensional structure, balancing the rotation and gravity effects. The size of the vortex soliton is from few up to few tens of light-days, depending on the black hole mass. Elongation of the vortex solution depends on the differential rotation and magnetic filed effects, which is equivalent to the viscous effect naturally involved into calculus. In the case of inner part of the galaxy, soliton solution provides an explanation of parameter regime that has to be satisfied according to observations. This solution can be used to develop improved dynamical model of accretion disk; it is expectable that nonlinear effects will influence dynamics and overcome some unresolved problems.

We have found that brief discussion on the viscous effects in the accretion disk theory is necessary. If there is rotation present in the model it needs to be differential rotation, meaning that different layers of the disk, dependent on the distance from the center, rotates with different speeds regarding the distance from the center of rotation So that, Keplerian rotation does not represents justified assumption, specially when self-gravity is incorporated into the dynamics. Differential rotation in that case represents natural viscous effect rising from the self-consistent approach.

It would be of interest to investigate the influence of other important parameters on the soliton solution, that have been neglected in this work (dissipation, radiation...) since this solution could be considered as a transient equilibrium. So that, soliton could be subject of further perturbation analysis. On the other side, it turns out that exact solution of Poisson's equation, with explicit functions for density and gravitational potential in the direction parallel to the angular velocity, has great importance on the disk stability. In the case of accretion disk, although the accretion disk theory is usually based on dynamics of particles moving in central gravitational field, neglecting self-gravity of the disk, the type of the solution derived in this work could be used to improve imposed gravitational potential. Even more, the potential variation can be subject of the equilibrium property of the disk that is possible to estimate using this type of solution. Detailed application of the nonlinear solutions on the accretion disks by comparison with observed spectra of the specific region will be subject of further research.

Acknowledgements

This research was supported by the Ministry of Science, Technological Development and Innovation of the Republic of Serbia (MSTDIRS) through contract no. 451-03-66/2024-03/200002 made with Astronomical Observatory (Belgrade).

References

Balbus, S. A. & Hawley, J. F.: 1998, *Rev. Mod. Phys.*, **70**, 1.
Chakrabarti, S. K.: 1995 *ApJ*, **441**, 576.
Chandrasekhar, S.: 1960, *Proc. Natl. Acad. Sci.*, **46**, 253.
Du, P. & Wang, J.-M.: 2023, *A&A*, **671**, A26.
Fine K. S., Driscoll C. F., Malmberg J. H. & Mitchell T. B.: 1991, *Phys. Rev. Lett.*, **67**, No. 5.
Gezari, S., et al.: 2007, *ApJS*, **169**, 167.
Hasegawa A., Maclennan C. G. & Kodama Y.: 1979, *Phys. Fluids*, **22**, 2122.
Lin D. N. C. & Pringle J. E.: 1987, *MNRAS*, **225**, 607.
Lin C. C. & Shu F. H.: 1964, *Astrophys. J.*, **140**, 646.
Lovelace, R. V. E. & Hohlfeld, R. G.: 1978, *ApJ*, **221**, 51.
Marcus P. S.: 1989, *Nature*, **331**, 693.
Papaloizou, J. C. B. & Lin, D. N. C.: 1995 *ARA&A*, **33**, 505.
Petviashvili V. I.: 1983, *Pis'ma Astron. Zh.*, **9**, 253.
Romeo A. B.: 1992, *MNRAS*, **256**, 307.
Shakura, N. I.& Sunyaev, R. A. : 1973, *A&A*, **24**, 337.
Smailagić, M. & Bon, E.: 2015 *J. of Astron. and Astrophys.*, **36**, 513.
Sniegowska, M., Czerny, B., Bon, E., et al.: 2020 *A& A*, **641**, A167.
Sommeria J., Meyers S. D. & Swinney H. L.: 1989, *Nature*, **337**, 58.
Vandervoort P. O.: 1970, *Astrophys. J.*, **161**, 67.
Velikhov, E.: 1959, *Soviet Phys, JTEP Lett.*, **36**, 1938.
Vukcevic M.: 2019, *MNRAS*, **484**, 3410.
Vukcevic M. & Popović L. Č.: 2020, *Nonlin. Processes Geophys.*, **27**, 295.
Yecko, P.: 1995, *Waves in Astrophysics*, **773**, 95-110.
Zakharov V. E. & Kuznetsov E. A.: 1974, *Sov. Phys. JETP*, **39**, 285.