Publ. Astron. Obs. Belgrade No. 103 (2024), 41 - 44 *Poster*
https://doi.org/10.69646/aob103p041

AN ALIASING METHOD FOR DETERMINATION OF TRANSPORT DATA FOR EXOTIC CHARGED PARTICLES IN CROSSED ELECTRIC AND MAGNETIC FIELDS

N. A. GARLAND[1], R. D. WHITE[2], R. E. ROBSON[2] and M. HILDEBRANDT[3]

[1]*School of Environment and Science, Griffith University, Nathan 4111, Australia*
E-mail n.garland@griffith.edu.au
[2]*College of Science and Engineering, James Cook University, Townsville 4810, Australia*
[3]*Paul Scherrer Institut, 5232 Villigen PSI, Switzerland*

Abstract. Understanding current and emerging particle physics experiments relies on quantifying the transport of exotic charged particles, such as muons. In some cases the fundamental scattering cross sections of exotic charged particles is not known, let alone the macroscopic transport data such as mobility or diffusion coefficients. As one path forward to remedy this dearth of data, we discuss an aliasing method that leverages the known transport data of well-studied ions to approximate the lacking, but much needed, transport of more challenging charged particles, such as positive muons as presented in this work.

1. INTRODUCTION

A good theoretical understanding of the behaviour of charged particles in gases is essential for modelling beam experiments. For some charged particles of interest, such as electrons, ions, or even positrons, there are well-established theory and experimental techniques to determine fundamental scattering cross sections and transport data to assist in modelling. For some exotic charged particles, such as muons, determining this transport data from theory or experiment is a more challenging task, and in some cases there may be no data at all.

This work presents an empirical approach to quantifying transport data of a beam of positive muons μ^+ in helium gas, subject to crossed electric and magnetic fields. A specific application of this scenario is central to the planned operation of the muCool experiment [Antognini *et al.*]. For understanding this experiment, one requires the input of (μ^+, He) transport data, which is, however, unavailable. A similar problem also arises in fundamental simulations, which may require the input of thus far unknown (μ^+, He) scattering cross sections. In this case, Taqqu recently argued that known (H^+, He) cross sections could be adapted to the muon problem for simulation purposes, and in a similar spirit we adapt or "alias" known (H^+, He) swarm experimental transport data as a method to approximate (μ^+, He) transport. Such a procedure has already been discussed in the context of momentum transfer theory for muons in an electric field only [Robson *et al.*], while this present work extends the method to crossed electric and magnetic fields.

2. METHODS

With a need for transport coefficients of exotic particles in interpreting experiments or prescribing input for theory or simulation, there is the obvious question as to what data could be used when no swarm data, or fundamentally calculated data from interaction cross sections, is available. Specifically, for muon beams in gases a previously discussed method of determining a muon reduced mobility in a background electric field [Robson *et al.*] through aliasing motivates the development of aliasing of swarm transport data presented here.

In this aliasing method, we seek to determine the mobility of a charged particle species, termed species (1), purely from knowledge of known mobility data of a surrogate particle, species (2), in the same gas. Here, we assume species (1) and (2) are of the same charge, and that the nature of the interactions between both species (1) and (2) and the background gas molecule are similar. For example, in this study we will turn attention to obtaining transport data of μ^+ in He given known transport data of a surrogate particle, H^+, in He [Ellis *et al.*]. Given that an introduction to aliasing swarm data in the absence of magnetic fields may be found in the work of Robson *et al.*, we will now briefly formulate the extension to the case of perpendicular magnetic and electric fields as relevant to the understanding of the muCool apparatus.

Critical to the aliasing procedure are assumptions about the collision dynamics in both the surrogate and desired systems. As outlined previously by Robson *et al.*, we assume that the elastic momentum transfer cross section (MTCS) of projectiles (1) and (2) evaluated at equivalent centre of mass (CoM) energies is the same,

$$\sigma^{(1)}{}_{m}\left(\varepsilon^{(1)}{}_{CM}\right) = \sigma^{(2)}{}_{m}\left(\varepsilon^{(2)}{}_{CM}\right). (Eq. 1)$$

To connect to previous MTCS scaling concepts in literature [see Krstić and Schulz, Senba] we note that equivalent CoM energies implies a scaled velocity by some ratio of reduced masses, i.e. so called velocity scaling as often termed in literature. In contrast, some authors have identified so called energy scaling, by a ratio of reduced masses, to be appropriate for other cross sections such as total elastic, or viscosity cross sections.

In the limit of zero magnetic field, invoking equivalent MTCS at a given CoM energy is sufficient to constrain the problem and deduce an aliasing relationship from one particle's transport data to another, via reduced mass ratio scaling. Here we consider the case of an orthogonal, non-zero **B** component. In order to limit the number of degrees of freedom in our problem, we must assume another constraint between the surrogate and desired scattering systems. Here, we assume equivalence of the Lorentz angles, ϕ, of particle (1) and (2), which are impinging upon a target atom with the same CoM energy. The Lorentz angle is defined as the angle, ϕ, between a particle beam's average velocity and the background electric field. This assumption is quite reasonable given the fixed orientation of electric and magnetic fields, and thus the Lorentz angle of a charged particle relative to a scattering target will be equivalent independent of what the particle is. With the two critical assumptions of equivalent CoM energy and Lorentz angle in our desired system (1) and a proposed surrogate system (2), we may then find that

$$\frac{K^{(1)}}{K^{(2)}} = \sqrt{\frac{\mu^{(2)}}{\mu^{(1)}}}. (Eq. 2)$$

Given a crossed field configuration we may invoke Tonks' theorem to propose $\varepsilon_{CM}(E,B,\theta) = \varepsilon_{CM}(E_{eff},0,0)$, and $W(E,B,\theta) = W(E_{eff},0,0)$, where an effective electric field in the sense of a conventional electric field only swarm experiment is given by

$$E_{eff} = E\sqrt{\frac{1+(\Omega/\nu_m)^2 cos^2\theta}{1+(\Omega/\nu_m)^2}}, (Eq. 3)$$

where θ is the angle between E and B, the cyclotron frequency is $\Omega = qB/m$. If the angle between E and B is $\theta = \pi/2$ we can simplify matters such that

$$E_{eff} = \frac{E}{\sqrt{1+(\Omega/\nu_m)^2}}. (Eq. 4)$$

Assuming that the Lorentz angle is fixed between charged particle (1) and (2), i.e. $tan\phi = K^{(1)}B^{(1)} = K^{(2)}B^{(2)}$, then it follows that

$$\frac{B^{(1)}}{B^{(2)}} = \sqrt{\frac{\mu^{(1)}}{\mu^{(2)}}}. (Eq. 5)$$

Making use of the Wannier energy relation, in the limit of low electron energies where inelastic collisions can be neglected, $\varepsilon_{CM} = \frac{3}{2}k_B T_g + \frac{1}{2}M\langle v\rangle^2$, after some algebra one can obtain

$$E^{(1)} = \sqrt{\frac{\mu^{(1)}}{\mu^{(2)}}}\sqrt{(E^{(2)})^2 + \frac{3k_B}{M}\left[\frac{1+(K^{(2)}B^{(2)})^2}{(K^{(2)})^2}\right]\left[T_g^{(2)} - T_g^{(1)}\right]}.$$

3. DEMONSTRATION OF ALIASING

Given the proposed mapping between a desired $E^{(1)}, B^{(1)}, K^{(1)}$ data set and a known data set for an ion, $E^{(2)}, B^{(2)}, K^{(2)}$ we can consider constructing a set of data based on given swarm experiment data for H^+ in He in only an axial electric field. If we invoke the effective field assumption, we may consider known swarm data of an (E, K) pair from a swarm experiment to be an effective field $(E|\quad|eff, K)$ that is then mapped to a possible $E^{(2)}, B^{(2)}$ orthogonal field pair, which then corresponds to a scaled reduced mobility, KN, surface for μ^+ in He as a function of local reduced fields. A surface plot of KN is shown in Figure 1 and demonstrates the utility and potential application of the proposed aliasing procedure in a crossed electric and magnetic field scenario for determination of positive muon reduced mobility in helium gas, where otherwise no prior data was available.

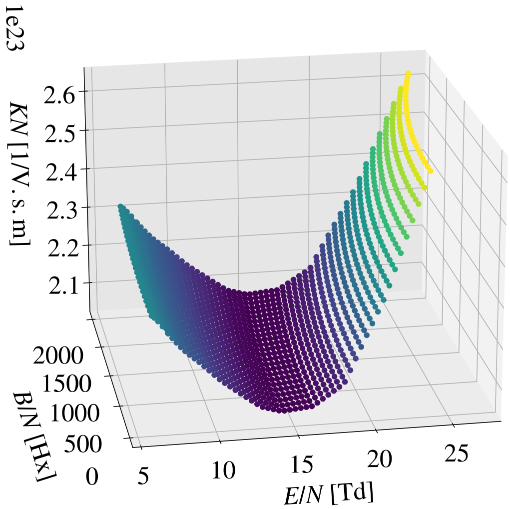

Figure 1 - Reduced mobility surface for μ^+ transport in He at 11 K, as a function of reduced electric and magnetic fields. Data generated by aliasing of swarm data measurements of H^+ in He [Ellis *et al.*].

References

Antognini, A. *et al.* (muCool Collaboration) : 2020, *Phys. Rev. Lett.*, **125**, 164802.
Ellis, H. W. *et al.* : 1984, *Atomic Data and Nuclear Data Tables*, **31**, 113–151.
Krstić, P. S. and Schultz, D. R. : 2006, *Phys. Plasmas*, **13**, 053501.
Robson, R. E. *et al.* : 2012, *J. Chem. Phys.*, **137**, 214112.
Senba, M. : 1989, *J. Phys. B: At. Mol. Opt. Phys.*, **22**, 2027.
Taqqu, D. : 2006, *Phys. Rev. Lett.*, **97**, 194801.